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ABSTRACT

This guide is intended as a source of ideas, procedures and suggestions for curriculum committees and developers wrestling with the planning, implementation, evaluation, and/or revision of kindergarten through grade 12 mathematics programs. It attempts to provide answers to questions which arise during the curriculum development process, and is divided into two major parts. The first, Developing a Mathematics Curriculum, contains: (1) An Overview; (2) The Mathematics Curriculum Guide; (3) Philosophy and Goals; (4) Design and Implementation; (5) Meeting Individual Student Needs; and (6) Student and Program Evaluation. Part II, Mathematics Curriculum Content, has: (7) The K-8 Program; (8) The 9-12 Program; and (9) Interdisciplinary Considerations. A large number of appendices provide detailed supportive information. (MP)

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A GUIDE TO CURRICULUM DEVELOPMENT IN MATHEMATICS

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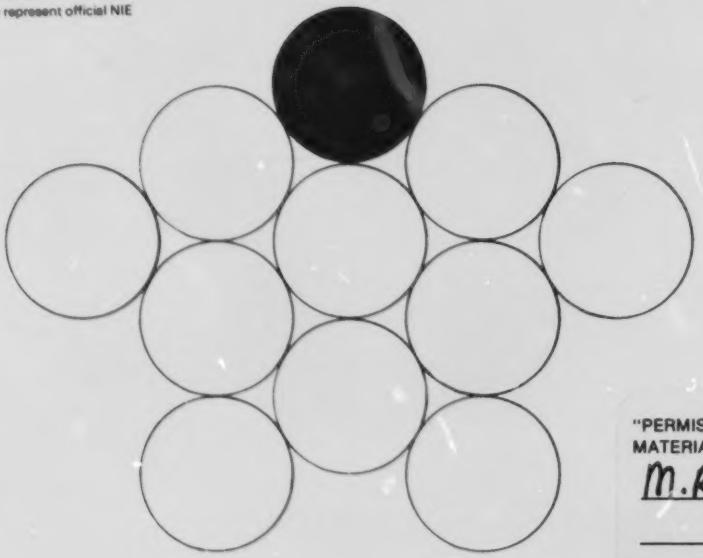

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This is one of a series of 12 guides to curriculum development prepared under the direction of the Bureau of Curriculum and Staff Development, Division of Elementary and Secondary Education, and published by the Connecticut State Department of Education. The guides may be reproduced in whole or in part as needed.

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Foreword

Connecticut has a strong commitment to equity and excellence in public education. The *Comprehensive Plan for Elementary and Secondary Education, 1980–1985*, embodies that commitment. Now this guide to curriculum development, part of a series, is one of the ways in which the State Board of Education is carrying out that commitment.

This concern for equal educational opportunity, dominant in the 1970s and continuing into the 1980s, has been expressed in a number of notable actions:

The State Supreme Court's historic school finance reform decision (*Horton v. Meskill*, 1978) led to Connecticut's educational equity legislation.

Statutes growing out of this concern for educational equity are Sections 10-262c, 10-262e and 10-16b of the Connecticut General Statutes. Sections 10-262c and 10-262e alter public school funding practices, more than doubling state support over a five-year period and setting a required minimum expenditure per pupil in each school district. Section 10-16b specifies educational programs which must be offered in all districts, with the requirement that they be "planned, ongoing and systematic."

In Connecticut's *Comprehensive Plan for Elementary and Secondary Education, 1980–1985*, submitted to the General Assembly in 1980, the State Board of Education pledged to offer local school districts a greater level of technical assistance and more positive leadership in planning, implementing and evaluating school programs.

The guides have been developed to provide tangible assistance and support to local school districts in complying with the legislative mandate. The titles of the guides correspond to the subjects which Section 10-16b requires all school districts to offer their students: the arts; career education; consumer education; health and safety; language arts, including reading, writing, grammar, speaking and spelling; mathematics; physical education; science; social studies, including, but not limited to, citizenship, economics, geography, government and history; and, at least on the secondary level, one or more foreign languages, and vocational education. The goals and objectives set forth in each of the guides relate to the

statewide goals endorsed in the *Comprehensive Plan*, namely, motivation to learn, mastery of the basic skills, acquisition of knowledge, competence in life skills and understanding of society's values.

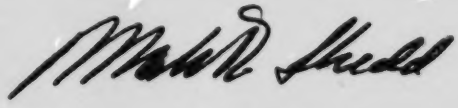
A Guide to Curriculum Development in Mathematics points the way toward a strong and effective mathematics curriculum for kindergarten through 12th grade. We view mathematics as a cornerstone of basic education and an essential component of each school's overall program. Mathematics is a "must" for those working in sophisticated research and development in medicine, electronics, energy, and the environment, for example, as well as for those seeking entry-level employment after high school.

Given the increasing and pervasive use of calculators and computers in our society, it is essential that students learn much more than the mechanics of computational skills. Students must also receive a strong foundation in mathematical concepts and problem-solving techniques in order to function effectively in our technological world. This guide will assist local school districts in developing a planned, ongoing and systematic program directed toward those outcomes.

The State Board of Education curriculum guides are not mandated courses of study for any student or any grade level. Each is intended solely to assist local district educators in the development of curricula. Each guide reflects the thinking and experience of an array of experts in its subject area who become, through this document, an important resource to local district educators.

The Connecticut State Board of Education frequently has expressed its conviction that the diversity of the state's public school system is one of its great strengths. Students, schools and communities do not have identical educational needs; imposing a standardized curriculum would impair, not improve, learning opportunities for students.

It is important for local district educators to keep the position of the Board in mind as they use this guide. There is much of value here which can be used to strengthen instructional practices and promote excellence in the curriculum development process. But these ideas can only enhance, not replace, the creativity, talent and commitment of the people in our local school districts who use this guide.



Mark R. Shedd
Commissioner of Education

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Finally the guide would not have been possible without the advice and substantive contribution of Clare Clark who brought to this document a sensitive understanding of how children do mathematics.

To all of these people, our sincere thanks.

Steven Leinwand
Consultant in Mathematics

Preface

A Guide to Curriculum Development in Mathematics is written to assist those persons in school districts who are involved in developing an educationally sound mathematics curriculum that is responsive to students, staff and the community. It is intended as a source of ideas, procedures and suggestions for curriculum committees and developers wrestling with the planning, implementation, evaluation and/or revision of kindergarten through 12th grade mathematics programs. It attempts to provide answers to questions which arise during the curriculum development process. Moreover, through the presentation of models and examples, this guide attempts to clarify the options available to mathematics curriculum designers.

More specifically, this guide is designed to help answer such questions as:

- What skills and experiences constitute a suitable program in mathematics for a local school district?
- What should an exemplary local mathematics curriculum guide accomplish?
- What decisions must be made with respect to the format of a local guide?
- What considerations must be given to implementing the curriculum successfully?
- What procedures should be used to evaluate the effectiveness of a mathematics program?

In some communities the "curriculum guide" is merely a localized mathematical topics index. In other localities, it is a more sophisticated organization of behavioral objectives. Some guides include specific activities, others include evaluation instruments, and still others relate objectives to specific instructional materials. Whatever the design presently in use or being considered for the future, school district personnel should find *A Guide to Curriculum Development in Mathematics* useful. It attempts to provide direction and offer alternatives that will facilitate the complex task of designing a responsive and effective mathematics program.

Part I
Developing a Mathematics Curriculum

An Overview 1

Throughout the past 30 or so years, mathematics education has been bombarded by change. Whether from societal forces applying pressure from the outside, professional judgments arising from within, or technological advances offering new and exciting opportunities, change and the inevitability of continued change make the development and maintenance of a mathematics curriculum both challenging and imperative.

A generalized overview of the recent past clearly demonstrates the breadth of this change.

The late 1950s saw the advent of "new math" programs. Commissions and study groups were established; new projects and programs were produced. The "new math" was perceived by many as being anything innovative, experimental, or simply different from the pre-1950 mathematics of the elementary and secondary school. The race was on to teach more mathematics at lower grade levels, and to stimulate greater understanding of mathematical concepts and structure at all grade levels.

The 1960s can best be viewed as the age of implementation. With the dawning of the "space age," textbooks began to incorporate new programs and teachers were trained in the use of new materials. Federal and local agencies poured money into crash programs and new topics and approaches were assimilated into traditional curricula. Theoretically, a reasonable and realistic plateau should have been reached, with nearly all children well prepared in mathematics and equipped to pursue further study of more advanced topics. However, the sixties witnessed some evidence of what many skeptics warned from the beginning: many children did not achieve at the levels demanded by the new program. Many parents found their children's homework incomprehensible, many teachers never really taught the "new" curriculum, and the pendulum began to swing again.

The 1970s brought an era of retrenchment accompanied by occasional forays into still newer areas. Out of the sixties came demands for more attention to the child who had not succeeded. We entered a period of social consciousness during which emphasis shifted to the underachiever, the slow learner and the

exceptional child. In light of declining test scores, the cry of "back-to-basics" arose. Mathematics curriculum planners returned to their drawing boards once again in search of a new balance between basic computational skills and other priorities. The seventies also witnessed the emergence of new issues, such as the role of career education in mathematics, the metric system, computer science and the proper place for calculators.

We begin the 1980s seeking the resolution of a range of these and other mathematical issues. The polarized debates over "new vs. old," "abstract vs. concrete" and "formal vs. intuitive" are no longer appropriate as we search for a balanced mathematics curriculum that addresses the immediate as well as the future needs of all of our students.

The balanced curriculum, in the form of a planned, on-going and systematic mathematics program, is developed around our answers to such recurring questions as:

- Why study mathematics?
- What constitute essential or basic mathematical skills?
- How can we provide for individual student differences?
- How do we create an appropriate interrelationship among computational skills, concept development and problem solving?
- How can the curriculum be organized and taught so that students see the essential unity and basic structure of mathematics?
- How do we incorporate available technology and appropriate research findings into our curriculum?
- How do we develop in all students a sense of discovery, of wonder, and of appreciation for mathematics, for its contributions and for its practical value for productive lives?

The answers to these questions lead us directly to the challenges of curriculum development in the 1980s.

To help school districts meet these challenges, this guide attempts to provide direction and stimulate thought about the whys, whats and hows of a mathematics curriculum. Some of the suggestions made will be obvious; others will be highly controversial. For example, a strong argument is made for a significant resequencing of the traditional K–8 program. Increased use of calculators is encouraged and the guide advocates a major shift of emphasis towards problem-solving skills. Moreover, the guide suggests the utilization of a comprehensive local curriculum guide format and an expansion of the traditional grades 9–12 course offerings. Suggestions are made for meeting the diverse needs of students through a variety of approaches and for effectively evaluating both programs and individual students.

Clearly, the changes of the past 30 years have not diminished the range of issues facing the mathematics curriculum developer. However, by reviewing the evolution of the present-day mathematics curriculum and by anticipating the future needs of our society, a mathematics curriculum responsive to today's issues and problems can be developed.

The Mathematics Curriculum Guide 2

A mathematics curriculum guide is a structured compilation of a school's or district's program for mathematics education. An exemplary guide

- assists in planning and implementing mathematics instruction;
- outlines a basic framework for what to do, how to do it and when to do it;
- provides appropriate goals and recommends means by which these goals are attained;
- establishes sequences both within and between levels;
- allows for flexibility, and encourages experimentation and innovation;
- promotes interdisciplinary approaches and the integration of curricula when appropriate;
- suggests methods of evaluating the entire program and its individual components, and
- provides a means for its own revision.

In written form, the curriculum is most commonly presented in the form of a guide that includes the goals, objectives, content, learning experiences and instructional resources that emerge from the curriculum development process. The formulation of such a guide should not be viewed as the culmination of the process, but rather as an essential way station along the path of ongoing curriculum development and implementation. Thus, no guide will be perfect; no guide will be free from criticism.

To be effective, however, a guide must earn acceptance by teachers and be deemed educationally valid. This will be significantly facilitated when the local curriculum guide is:

- consistent with what is known about child growth and development;
- compatible with the general philosophy of the school system;
- based upon positive convictions in response to recognized needs;
- integrated from kindergarten through 12th grade;
- readily usable by all teachers of mathematics;
- capable of flexibility in use, and
- cooperatively prepared.

A curriculum guide in any discipline is the result of a carefully planned process that integrates the efforts of many. The decision to proceed with curriculum development requires commitments by the local board of education and the school administration to provide the support and resources necessary to carry out the process.

Resources

Among these essential resources are the human resources that comprise and support a mathematics curriculum committee. Such a committee, consisting primarily of teachers representing the various schools and grade levels in a district, including administrators, members of the public and perhaps students, and led by an effective chairperson, becomes the driving force of curriculum change.

Time is also essential—time for people to think, to research and to write. After-school, evening or weekend meetings often are not productive. Effective curriculum development requires released time or summer work or a combination of both. Half-day or full-day released-time meetings on a biweekly or monthly basis can be productive, but frequently require either individual or subcommittee work between meetings. Summer work has proven successful because participants can devote their complete attention to the task. In either case, considerable time is required to do the job completely and correctly.

Further, a mathematics curriculum guide should draw upon a range of materials when and where appropriate to meet local district goals and needs. Among the source materials available are:

- the curriculum guide and textbooks presently in use;
- curriculum guides and scope and sequence charts from other school districts and states;
- selected textbooks, textbook series, workbooks and other nontext instructional materials;
- a hierarchy of skills based on developmental stages;
- professional journals (particularly the *Mathematics Teacher*, the *Arithmetic Teacher* and the *Journal for Research in Mathematics Education)*, and
- selected professional books, national curriculum studies and research reports.

These resource materials should be available to the mathematics curriculum committee. For an extensive list of mathematics curriculum development resources, see Appendix C.

Finally, adequate funding is also essential to the curriculum development process. Funds may be needed for items such as:

- salaries for school staff who participate in the work or for substitutes covering for released staff;
- cost of secretarial services and duplication of materials;
- purchase of materials, professional literature and equipment;
- fees for consultant services;

- travel expenses to visit other school systems;
- printing sufficient copies of the final guide, and
- costs of in-service training needed to implement the guide.

Forces that influence curriculum development

The decisions, attitudes, actions and perceptions of many individuals and groups exert pressures that influence curriculum development. These forces are exerted at the local, state and national levels. Although local forces, being closest, often have the strongest influence on school curricula, the impetus for change seeps down from other levels as well. An informed assessment of need and a sound process of curriculum development require an awareness of and a response to these forces and trends.

At the local level:

- **Students'** present and future mathematical needs must form the basis of a curriculum. For example, curriculum decisions involving the use of calculators or the emphasis on the metric system depend heavily upon an analysis of students' needs.
- **Teachers'** training, experience, attitudes toward change, and commitment to the curriculum development process significantly influence the planning and implementation of any curriculum.
- **Administrators'** support, leadership and enthusiasm have a direct effect upon the quality of school curriculum.
- **The school board's** policies, attitudes and allocation of funds directly influence curriculum
- **The local community's** expectations, interest in its schools and level of support affect a local curriculum.

At the state level:

- **The legislature,** by the appropriation of monies and enactment of legislation, fosters certain programs while subordinating others. The Education Evaluation and Remedial Assistance (EERA) Program, for example, affects curriculum directly through mandated provision of remedial assistance.
- **The State Board of Education** influences curriculum by the emphasis it places on particular programs, through statewide assessment programs and by the financial and personnel assistance it can offer. In addition, its policies help to shape the determination of local priorities.
- **Institutions of higher education,** by their entrance requirements, the availability of their staff as resources for local schools, and especially the quality of their teacher education programs, affect the local curriculum.
- **The business and industrial community,** by its entry level requirements, its increasingly technological orientation and its commitment to cooperative programs with schools, influences the nature of curriculum.
- **Professional organizations,** by providing opportunities for communication between local educators and leaders in the field through workshops, institutes and publications, influence local school personnel directly and the curriculum indirectly.

At the national level:

- **National curriculum studies and reports** have provided models and recommendations of importance to local curriculum developers. Such reports from the National Advisory Committee on Mathematics Education (NACOME, 1975), the National Assessment of Education Progress (1973, 1979) and the National Council of Teachers of Mathematics (*An Agenda for Action*, 1980) have addressed the present situation and encouraged a range of mathematics curriculum changes.
- **Textbook publishers,** because of the widespread use of their products, directly affect curriculum. The pervasive use of textbooks as the sole determinant of curriculum frequently limits or even removes decision making from the local level. This reliance on the textbook often leads to a mathematics program overwhelmingly based on paper and pencil tasks, devoid of essential activities entailing manipulative materials.
- **Standardized tests,** by their widespread use for student and program evaluation, also influence curriculum, particularly by subordinating those mathematics skills that are not readily measured on a multiple-choice instrument. When such tests are used as an evaluation measure, they strongly influence the curriculum.
- **Federal programs,** including institutes, model projects and curriculum studies, can provide additional impetus for the development of curriculum. The wide dissemination of model Title I and Title IV-C projects involving mathematics and the availability of funding for the purchase of computers and other materials attest to the possible influence of federal programs.
- **The general public,** by its demands for programs that address society's problems, affects curriculum. The clamor following Sputnik, and more recently, attention to poverty, illiteracy, testing and "back to basics" are examples of how the public at large influences curriculum, particularly in a basic skill area like mathematics.

Recognition of these forces enables the mathematics curriculum developer to create an effective, responsive and useful curriculum.

Assessing needs

However, these forces must be reviewed in conjunction with a determination of the need for change within a local school district. Such a determination usually arises out of an assessment of the present situation.

- Teachers may be dissatisfied with older content and techniques in light of recent research.
- Test scores may be declining.
- Teachers and others may wish to relate the content of the mathematics program more closely to contemporary problems and circumstances.
- Teachers and administrators may wish to build greater accountability into the process of mathematics instruction.
- Students may express a desire for different curricular opportunities.
- Parents and others may suggest a reexamination of the curriculum due to dissatisfaction with the mathematics program or in light of social, technical and economic changes in the community.

Whatever the particular circumstances, an initial decision of whether to proceed with curriculum development must be made. If the program is functioning well and meets the apparent needs of students, staff and community, the decision may be that no change is warranted at present. If, however, the program is deemed to be in need of improvement or updating, a decision to initiate the curriculum development process is made.

At this point, the question becomes, "What changes are needed?" A district may decide upon:

- complete revision of its K–12 mathematics program;
- revision of a large segment of the program, such as the K–6, K–8 or secondary program, or
- modification of an individual part of the present program, such as a revised program for slow learners at the middle school level or a new elective at the secondary level.

Although a complete curriculum revision may be undertaken, the new program usually will include components of the present program that are effective and that provide the foundation upon which changes are built.

In identifying needs, one model is:

Desired Outcomes	minus	Current Status	equals	Needs

In this equation, curriculum development can be viewed as a process by which the meeting of needs leads to improvement of the current status to arrive at desired outcomes.

Regardless of the theory or model followed, curriculum developers should gather as much information as possible on desired outcomes or expectations of a mathematics program, as well as the current status of student achievement, actual program content, and the concerns and attitudes of teachers, administrators, parents and students.

This information is most frequently gathered through surveys, discussions, test data and professional literature.

- Mathematics teachers may analyze the present curriculum guide to identify strengths, weaknesses and/or problems.
- Teachers at each grade level may identify what they believe to be major problem areas in the mathematics curriculum.
- Recent student test data, particularly at the item level, may be analyzed.
- Meetings of teachers and administrators may generate suggestions for the improvement of the mathematics curriculum.
- Parents and other members of the community may be surveyed or invited to meetings to ascertain their interests, expectations and perceptions of student needs.

- Curriculum guides from other school districts and from various state departments of education may be examined.
- Consultants may provide a critique of the current mathematics program.

These collected data must then be correlated with the curriculum, textbooks, supplementary materials and standardized tests currently in use to identify those areas where changes are required to improve the program.

The ultimate usefulness of the curriculum depends greatly on the attention given to identifying the major forces and needs and then responding to them.

3 Philosophy and Goals

The fundamental questions that the developer of a mathematics curriculum must answer are "Why mathematics?" and "Upon what guiding principles should our mathematics program be built?" The answers to these questions form the philosophy and goals of the program and provide the unifying framework that justifies and gives direction to all mathematics instruction.

Having studied curricular trends and assessed the present program, a curriculum developer begins to construct a guiding philosophy of instruction and a set of educational goals for the mathematics program. The philosophy and goals often reflect the uniqueness of each school district and the diverse needs of students served.

A sound mathematics curriculum includes a stated philosophy of mathematics education. Such a philosophy or set of beliefs helps to determine the scope of a curriculum as well as the materials and techniques that are used to implement it. A philosophy needs to be more than just "what we think should be happening," but rather "what our curriculum is actually striving to reflect."

Sample philosophy statements are presented here without elaboration.

- We believe that students must be active in the learning process.
- We believe that the most important goal of mathematics instruction is the development of the students' ability to solve problems.

- We believe that, since it is hard to determine the specific skills that will be needed in the future, it is important that students learn how to learn and develop a willingness to continue learning.
- We believe that students mature cognitively at different rates.
- We believe that students respond to differing kinds of motivation.
- We believe that students respond to different styles of teaching.
- We believe that students have different learning styles.
- We believe that since each student is unique, we cannot justify the *a priori* notion that all students should reach a fixed level of mathematical achievement. Further we believe that mathematical experiences should be available to every student at his/her own level of ability.
- We believe that mathematics education must not remain isolated from other curricular areas or disciplines.
- We believe that mathematics instruction should be a judicious blend of concrete and abstract, application and theory, skills and concepts.
- We believe that students should enjoy the study of mathematics.

Like a philosophy statement, mathematics program goals arise from the conceptualization of the value and purposes of mathematics instruction and knowledge. As such, appropriate goals might recognize mathematics as:

- a means of expressing quantitiable ideas;
- training for discipline of thought and for logical reasoning;
- a tool in activities arising from the needs of engineering, technology, science, economics, psychology, medicine, business, agriculture, and other professions, and
- a study in itself, where development of new techniques and concepts expands knowledge, understanding and appreciation.

The following statements are examples of goals for mathematics programs. Although several of the suggested goals overlap, the reader is encouraged to choose or adapt those goals that are appropriate to local needs.

To develop in each student

- a conceptual understanding of mathematics;
- the skills of critical thinking, inductive and deductive reasoning, and problem solving;
- appropriate computational skills;
- the ability to apply mathematics to problems of everyday life, vocations and careers;
- the ability to use mathematical tools and concepts for measurement and problem solving;
- an appreciation for and understanding of mathematical patterns and structures;
- an appreciation for logical processes and the ability to justify rational decisions, and
- confidence in and a favorable attitude toward mathematics.

Additional mathematics program goals might be to:

- provide a curriculum that reflects varying student abilities and interests;
- provide students with exposure to interdisciplinary knowledge;

- provide a perspective for recognizing, formulating and analyzing problems, for judging and working out strategies and for verifying results;
- establish a technical basis for the social and physical sciences;
- contribute to an appreciation of the beauty and power of mathematics and the excitement of the challenge of mathematics, and
- strengthen self-reliance and independent thinking and encourage the flexibility and versatility needed to cope with a highly technological society.

Goals such as these help to establish a foundation for the entire mathematics program and form the basis for the subsequent development of mathematics courses and student objectives. Moreover, these suggested program goals strengthen and amplify the statewide goals for education stated in the Connecticut *Comprehensive Plan for Elementary and Secondary Education, 1980–1985*, and listed in Appendix A.

4 Design and Implementation

The design and organization of a curriculum guide are key determinants of its ultimate usefulness. The basic content question—"What is to be learned?"—soon raises other basic questions.

- In what form should the content be presented?
- In what sequence should the material be taught/learned?
- What instructional techniques and materials should be used to foster the teaching/learning?
- What means should be used to evaluate the teaching/learning?
- How can these design considerations be incorporated into a guide?
- How can the curriculum be implemented?

Objectives and format

There is obviously no one correct way to organize the content that is outlined in a curriculum guide. Frequently a guide provides two or more interrelated formats of the material, such as a scope and sequence listing of topics or objectives by content strands in one place and a grade by grade listing of objectives in another.

Whatever the specific format, a guide must present the content in the form of specific objectives. Such "topic" statements as "multiply whole numbers" at grade 3 or "solve equations" in an algebra course are often far less useful than specific objectives such as "Given a 1-digit factor and a 2-digit factor, the student will find the product" or "Given a linear equation involving several operations, the student will solve it using the appropriate properties correctly."

Such specificity, often lacking in a curriculum guide, provides both the student and the teacher with a clear description of what must be learned and taught. Further, specific objectives facilitate teaching and the creation of appropriate assessment instruments to measure the success of that teaching.

An effective and comprehensive curriculum recognizes the need for balance among cognitive, affective and psychomotor objectives. Since paper and pencil tests assess primarily cognitive objectives, curriculum developers often subordinate the affective and psychomotor domains. The multiplication and equation-solving objectives noted above reside within the cognitive domain because they relate to mental processes such as knowing, comprehending or analyzing. They are examples of the type of objective with which educators are most familiar and comfortable.

Objectives from the affective domain might include:

- Given a nonroutine problem (i.e., a problem without an obvious procedure, but appropriate to the student's ability level) the student enjoys trying to solve the problem.
- Given a measurement problem involving the conversion of measurement in both customary and metric units, the student appreciates the greater simplicity of using metric units to solve the problem.
- Given a problem involving the application of similar triangles, the student appreciates the practical value of geometric skills and concepts.

Objectives from the psychomotor domain might include:

- Given a ruler or tape measure and an object, the student will measure the object to the nearest centimeter or half inch.
- Given a set of number blocks, the student will represent a 3-digit number with the blocks.
- Given a straight edge and compasses, the student will construct the perpendicular bisector to a given line segment.

Objectives from the affective and psychomotor domains are often difficult to incorporate into a curriculum guide. Moreover, they are often more difficult to measure than cognitive domain objectives. However, these recognized constraints should not result in the omission of such objectives from curriculum guides.

In selecting and writing objectives, several questions should be asked.

- Is the objective sufficiently specific to give the reader a clear picture of what the student should be able to do, without being so detailed as to make the statement labored or the objective trivial?

- Is the objective compatible with the goals and philosophy of the program and the needs of the students?
- Is the objective realistic and attainable by students?
- Are appropriate materials and resources available to make the objective achievable?

As objectives are selected and written, they must be organized in an orderly fashion. This order can be achieved in numerous ways: by grades, by strands, in units, in sequential levels of instruction or through some combination of these. Decisions on the organization of a guide must be made carefully and must reflect the agreed-upon philosophy of mathematics education and the preferences of teachers who are to use the guide.

- A *graded* structure organizes objectives by the grade which a student is in and is the most commonly used structure.
- An organization by *units* groups objectives by main topics. Units may or may not be of differing difficulty and may be large or small, sequential or nonsequential. A unit organization is most commonly used for high school courses.
- A *strand* organization places all of the objectives for a specific topic or strand together in a sequential order, without regard to specific grade. Such an organization lends itself to individualized instruction and continuous progress within a strand.
- A *sequential* organization outlines objectives in a continuous chain without regard for grade level or strand, and allows for individual student progress along a continuum of skills and experiences.

Often, an effective guide will incorporate more than one format. For example, a common arrangement lists objectives grouped by strand within each grade level. In this manner the 3rd grade teacher is provided with a complete listing of the 3rd grade objectives organized by strand or major topic. However, it is important for this teacher to have access to the 2nd grade objectives containing skills that may have been introduced, but not taught for mastery, as well as forthcoming 4th grade objectives. This information is often provided in a scope and sequence listing by strand that would place the 3rd grade measurement objectives, for example, in the context of the entire K–8 measurement strand.

These various formats are illustrated on the following pages. Table 1 illustrates a sample scope and sequence chart for subtraction of whole numbers. Table 2 illustrates a sequential organization of measurement objectives for grades K–5, and clarifies each objective with an example. Table 3 illustrates some specific student objectives for fractions at the 5th grade.

Table 1
Content Area: Whole Numbers–Subtraction

Topics	K	1	2	3	4	5	6	7	8
Subtracting a 1-digit number from another 1-digit number.		M							
Subtracting 2-digit numbers without regrouping (no borrowing).			M						
Subtracting a 1-digit number from a 2-digit number less than 20.				M					
Subtracting two 3-digit numbers without regrouping (no borrowing).				M					
Subtracting 2- and 3-digit numbers where tens are regrouped to get more ones (borrowing from tens only).				M					
Subtracting two 3-digit numbers where hundreds only are regrouped (borrowing from hundreds only, not from tens).					M				
Subtracting two 3-digit numbers where both tens and hundreds are regrouped.					M				
Subtracting two 4-digit numbers where both tens and hundreds are regrouped.					M				
Subtracting two 4- and 5-digit numbers (regrouping all the way).						M			

The letter "M" indicates the grade at which a topic is normally covered to the point of student mastery.

Source: East Hartford (CT) Public Schools Mathematics Curriculum

Table 2
Measurement Performance Objectives

Objectives	Examples
1. Compare two objects as to specified characteristics (length, weight/mass, volume, temperature).	1. a) Which stick is longer? b) Which is heavier, the ball or the balloon? c) Which glass of milk has more in it (same size glass)? d) Which is colder, the piece of ice or the pencil?
2. Use informal units of measurement to describe characteristics of a given object (length, weight, volume, temperature).	2. a) What is the width of the desk in pencil lengths? b) How many glasses of Kool-Aid are in this pitcher? c) Tell me how many nickels this chalkboard eraser weighs.
3. Identify order of three objects as to specified characteristics (length, weight, volume, temperature).	3. a) Which of the three boys is tallest? b) Tell me which book is heaviest. c) Which container has the most water? d) In what season are the days the hottest?
4. Recognize place relationships, speed comparisons.	4. a) Which of two trees is farther from us? b) Which of two cars is traveling faster?
5. Identify coins and value (penny, nickel, dime).	5. a) Show me the penny. b) Show me the nickel. c) Show me the dime. d) Which of the three coins is worth the most?
6. Recognize the need for standard units of measure by making simple measurements using non-standard units.	6. a) How many steps long is the classroom? b) Use your hand span to measure the width of the chalkboard.
7. Make simple linear measurements to the nearest unit in centimeters, decimeters, meters, inches, feet and yards.	7. a) How long is this pencil to the nearest centimeter? b) How long is the playground to the nearest yard?
8. Tell time to the nearest hour and half hour and name seasons, months and days of the week.	8. a) Large calendar with removable month names is brought up to date each day, and future plans are made with its help. b) It is about time for lunch. Where is the small hand on the clock when we come to school? When we leave to go home? Egg or oven timer can be used to mark time intervals.
9. Tell time by season, month, and hour.	9. a) Look at the calendar and tell me what day it is. b) What season immediately follows spring?
10. Tell temperature in degrees (Celsius and Fahrenheit).	10. To the nearest degree, what is the temperature on the thermometer?

11. Measure the weight (mass) of an object to the nearest kilogram and pound.
12. Measure capacity in liters, cups, pints, quarts and gallons.
13. Compare the following units:
    a) cm, dm, m
    b) in., yd., ft.

14. Recognize that the time represented between succeeding numerals on a clock represents a five-minute time interval.
15. Measure length to the nearest half-inch.

16. State the value in cents of a penny, nickel, dime, quarter, half dollar, dollar.
17. Tell time to the nearest minute.
18. Convert given measures to equivalent measurement system.
19. Choose an appropriate measurement unit for a given situation.
20. Measure length to the nearest quarter-inch or centimeter.
21. Add and subtract measures without regrouping.

22. Measure lengths to the nearest sixteenth-inch and to the nearest millimeter.
23. Add, subtract and multiply measures with regrouping.

11. Look at the scale and tell me how many kilograms or pounds Billy weighs.
12. To the nearest liter, how many liters of water are in this bucket?
13. a) There are __ centimeters in a decimeter.
    b) There are __ decimeters in a meter.
    c) There are __ inches in a foot.
    d) There are __ feet in a yard.
14. How many minutes have gone by when the minute hand has moved from 12 to 4 on the clock?

15. Use rulers marked to the nearest quarter-inch to measure objects. Measure any line segment to the nearest half-inch.
16. How many one-cent bubble gums can I buy for a nickel? For a dime?
17. Look at the clock and tell me what time it is.
18. 5 km = 5000 m
    10,560 ft. = 2 mi.
19. What unit would you use to measure the distance around your head?
20. Measure any line segment to the nearest quarter-inch or centimeter.
21. If one piece of rope is 3 feet 4 inches long, and another piece is 2 feet 2 inches long, what is the difference in their lengths?
22. Draw a line segment on your paper. Now measure it to the nearest 16th-inch and to the nearest millimeter.
23. If I have a piece of wood 4 feet 9 inches long, and a second piece of wood 3 feet 5 inches long, how long are the two pieces together?

$$\begin{array}{r} 4'\ 9'' \\ 3'\ 5'' \\ \hline 8'\ 2'' \end{array}$$

Source: *Guidelines for Mathematics Instruction*, Indiana Department of Public Instruction

Sequencing

The pyramidal nature of most mathematics skills, building upon previously mastered skills and concepts, makes the ordering of specific objectives a crucial consideration. Further, the close interrelationship between stages of cognitive development and the effective learning of mathematics also demands a careful scrutiny of sequencing.

The grade placement of objectives has generally been dictated by tradition, textbook publishers, mathematicians and professors of education. However, a local curriculum guide provides the opportunity to adjust the order of content to meet specific local needs and the experience of local classroom teachers.

In developing a sequence for mathematics objectives and experiences, consideration must be given to two major issues.

First, the sequencing must recognize the need for introduction, conceptual development, mastery and retention or maintenance of skills. The mere presence of an objective at grade 6 is no guarantee of its mastery by grade 7. Also, it is often appropriate to introduce new skills or concepts at one grade level to anticipate what is to come, without attempting to develop mastery of the skill.

Table 3
Student Objectives–Fraction Strand—Grade 5

Given a number with up to three digits, the student will tell if it is divisible by 2, 3, 5, or 10.

Given models and examples (e.g., fraction pies, ratio of boys to total students in a class, what fractional part of a dollar is a dime, etc.), the student will identify the appropriate fractions.

Given a proper fraction, the student will name five equivalent fractions.

Given up to four different proper "ruler" fractions (i.e., having denominators of 2, 4, 8, or 16), the student will arrange them in order.

Given any two fractions (including mixed numbers) the student will determine whether or not the fractions are equivalent, or which is greater, and will use the correct symbol (<, =, >).

Given a fraction (having a denominator of 50 or less) not expressed in lowest terms, the student will reduce it to lowest terms.

Given an improper fraction, the student will rename it as a mixed number and vice versa.

Given two fractions or mixed numbers, the student will find the sum and express it in lowest terms.

Given two fractions or mixed numbers, the student will find the difference and express it in lowest terms.

Given a whole number up to two digits and proper fraction, or given two fractions, the student will find the product.

Source: Middletown (CT) Public Schools, Curriculum Standards: Mathematics

Further, many skills need to be reinforced, reviewed or practiced in the years after they are first taught. These realities require a spiraling of curriculum objectives wherein the same or similar objectives are taught at more than one grade level to assure mastery of prerequisite skills and concepts prior to the introduction of subsequent and related ones. This spiraling can be seen within the fractions strand, for example, where in the primary grades students master $\frac{1}{2} + \frac{1}{2}$, progressing to problems like $\frac{2}{3} + \frac{4}{5}$ in the intermediate grades, a/b + c/d in algebra and sin a/cos a + cos a/ sin a in trigonometry. Each case of addition of fractions depends upon mastery of the previously encountered skills.

Second, the sequencing of specific objectives and learning experiences must take into consideration our prevalent theories of learning. Robert Wirtz, in *Mathematics for Everyone*,[1] combines the research of Jean Piaget and Jerome Bruner into a single learning theory matrix (see Figure 1). The theories underlying the matrix suggest how learning takes place.

> All ideas a child fully internalizes must be met first in the world of three dimensional things—the manipulative world. Later, the same ideas can be encountered and understood in sketches and pictures and diagrams—the representational world. Eventually, the child can work with these same ideas expressed in symbols—the abstract world.

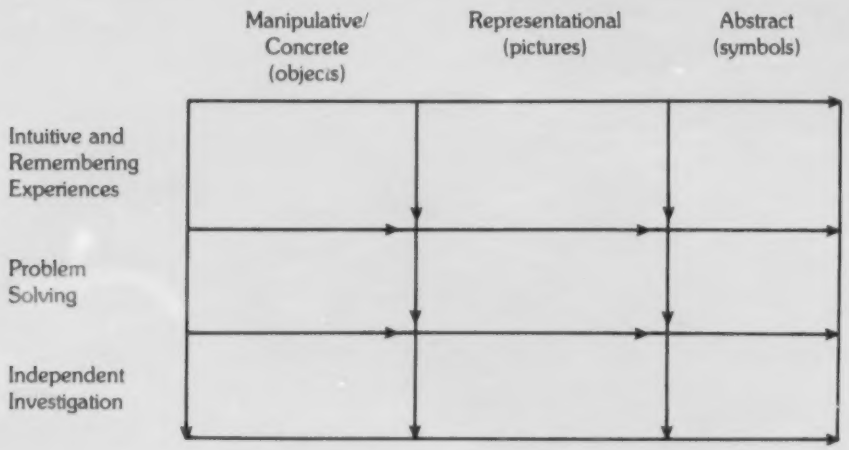


Figure 1
Learning Theory Matrix

> All learning begins as the senses bring a myriad of data to mind. Some data are retained, and the child's memory bank grows—he is remembering experiences. Soon, a child can use these experiences to determine relationships that are not obvious at the outset—he is solving problems. Eventually, the child is able to pose problems and work toward solving them—making independent investigations.

Movement through the matrix should be both from left to right and from top to bottom. Mathematics instruction often jumps to the "Remembering-Abstract" block of the above matrix with insufficient attention to concrete and pictorial experiences and inadequate attention to the subsequent use of these experiences to solve problems. In developing and appropriately sequencing objectives it is essential to provide these types of experiences. For example, in developing the concept of fraction, at the concrete level, students should explore cutting or folding sheets of paper; at the representational level, students should work with fraction bars and pictures of partitioned sets of objects; and at the abstract level, through these activities, students will gain an understanding of ½, ⅓, ¼, etc., all in an intuitive and remembering mode. Then the process would be repeated using paper, fraction bars and the fraction symbols to develop the concepts of nonunit fractions and equivalent fractions, building to rules for adding fractions with like, and then unlike, denominators. Instead of stopping with the rule for adding fractions, a well sequenced curriculum next applies the skills through problem solving and adds enrichment activities such as work with the tangram to provide students with the opportunity for independent investigation.

Similarly, in developing the concept of area, students might count square tiles that cover rectangular regions, explore rectangular regions on graph paper, and then arrive at the formula for the area of a rectangle ($A=L \times W$). This concept and skill can then be applied in the same progression in a problem-solving mode to determine the area of common rectangular classroom objects and then nonrectangular regions.

This developmental pattern is as valid and important in developing and sequencing many objectives for algebra or geometry, for example, as it is at the elementary level. A topic such as solving linear equations might be introduced by manipulating black boxes that represent variables and sticks that represent numbers, progressing to pictures of boxes and numbers on a balance scale and finally moving to the traditional (and abstract) $ax+b=c$ algebraic form.

Instructional techniques and materials

Most curriculum guides provide objectives that spell out *what* should be taught/learned, leaving the *how* it might be taught to the discretion of individual teachers. However, it should be clear from the above discussion that the effective teaching/learning of specific objectives entails a range of activities or learning experiences that respond to the "how" question and that do not readily lend themselves to written objectives.

For this reason, *an effective mathematics curriculum guide must provide more than just objectives.* This can be accomplished by supplementing objec-

tives with page notations or activities from the textbook, the teacher's guide for the textbook, and other resources. In some cases, where appropriate activities are not available in the text, the guide might include such activities. Figure 2 provides an example of such a page where, for a particular objective, the guide provides important information on prerequisite skills, materials and sample activities.

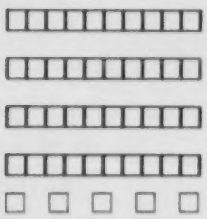
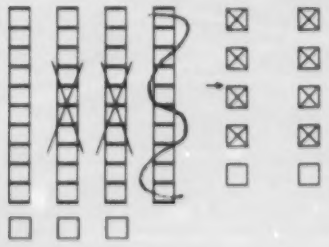
• Given a problem involving the subtraction of a 1-digit number from a 2-digit number, or a 2-digit number from a 2-digit number, the child will solve the problem using regrouping.	Objective
• The child must be able to write 1- and 2-digit numbers in vertical form and understand the exchange principle.	
• Squared materials (squares and strips), scissors	Prerequisite
	Materials
• The teacher presents an oral problem: 45 − 27 = . The child is given 4 strips and 5 squares. The child will remove 2 strips and 7 squares. The child cuts up a strip to show that a group of ten can be broken into units.	Activity (Concrete)
• The teacher may choose to limit numbers. Cards may be made with pieces pasted down so the child can cross out the amount to be subtracted. Continue the activity by supplying the children with various problems to be solved with the squared materials.	Reinforcement/ Extension
• Teacher-made cards with acetate top sheets.	Materials
• The child is given an oral or written problem which must be solved by crossing out the number to be subtracted from the picture represented. The child uses X to cross out what is subtracted and a squiggly line ~~ to represent what was regrouped. Example: 43 − 28 = □	Activity (Representation)
	
• Repeat this activity as shown using various equations. Have children make up equations for partner to solve.	Reinforcement/ Extension

Figure 2
Subtraction of 1- and 2-Digit Numbers with Regrouping

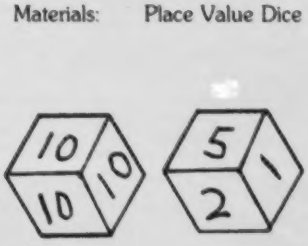
Source: Stamford (CT) Public Schools K–4 Curriculum Guide

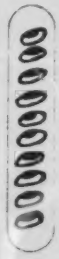
Evaluating teaching/learning

The issues of evaluating curriculum will be addressed in a later section. However, in designing curriculum it is essential to ask, "How will we know that the objective has been achieved?" A mathematics curriculum guide may suggest specific examples or it may suggest particular exercises from the text. Further, it should note those objectives that will be assessed on district-wide criterion-referenced tests or on locally used standardized tests. While the evaluation of mathematics objectives is usually in the form of a paper and pencil response, some objectives (e.g., measurement) should be evaluated in nonpaper and pencil ways.

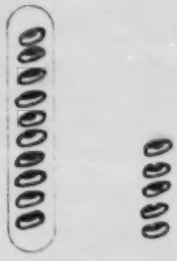
Grade: 1–2
Unit/Strand: Numeration/Place Value
Objectives: Construct, read and write numbers from 11 to 19
Learning Experiences/Activities:

Materials: Place Value Dice 10-Bean Stick Loose Beans





Procedure: Throw dice. Represent the number with a 10-Bean Stick and loose beans. **Always place 10-Bean Stick on the left.** Make a drawing to represent the number. Write the number in expanded form. Write the number. **Explain that the digit 1 represents the 10-Bean Stick and the other digit represents the loose beans.**



10 + 5 = 15

Resources: Place value dice, bean sticks, loose beans
Teacher's guide, p. 143–4.
Evaluation: Student correctly completes worksheet.

Figure 3
Curriculum Guide Format
(Sample)

Incorporating design elements into a guide

As curriculum developers broaden their conceptualization of written curricula and curriculum guides beyond mere listing of topics or objectives, design and format considerations assume greater importance. One possible format is shown in Figures 3 and 4. While time-consuming to develop, such a comprehensive format is sufficiently detailed to provide clear directions and understandings. Moreover, such a format allows a district to establish its own tailor-made program instead of letting the textbook dictate the entire curriculum.

Grade: 4–5
Unit/Strand: Measurement
Objective(s): Identify linear equivalents (customary units)
Convert linear measurements (inches, feet, yards)
Learning Experiences/Activities:

1. Use measuring tape to measure and graph waists and heights of classmates in inches.
2. Use one-foot rulers and five-foot measuring tapes to show relationship between feet and inches.
3. Use one-foot rulers and yardsticks to show relationship between feet and yards.
4. Have students use measuring tools to complete examples such as:

17 inches = ____ft.____in.

4 feet = ____inches

7 feet = ____yards____ft.

5. Have students place strips of masking tape on the floor and working in small groups, estimate and then measure their lengths in various units.

Resources: Tape measures, foot rulers, yardsticks, masking tape

Evaluation: 1) Completion of worksheet from activity 5 above

2) Textbook, page 251, examples 1–15

Figure 4
Curriculum Guide Format
(Sample)

Depending upon the degree of specificity, typical mathematics curricula consist of between 25 and 45 objectives in 2nd grade, between 50 and 75 objectives in 6th grade, and between 40 and 75 objectives in an algebra course. These data suggest the feasibility of employing a format similar to that shown in Figures 3 and 4 without unreasonable effort. Further, this format is used in Appendix D to provide samples of suggested curriculum content.

A less comprehensive, but still effective, curriculum guide format is shown in Figure 5.

Grade 7

Strand	Objectives	Examples	Resources	Evaluation
Ratio, Proportion Percent	Given two sets of quantities, the student will express the comparison as a ratio.	ratio of males to females; adults to students; cars to trucks; shaded to unshaded squares on graph paper	student-generated comparisons Text p. 235 graph paper	Text p. 236
	Given a ratio, the student will name two equivalent ratios.	1 to 3 is equivalent to 2 to 6 and 3 to 9	Teacher's guide p. 240–1 Text p. 240	Text p. 241 Worksheet
	Given two equivalent ratios, student will express them as as a proportion	2 to 5 and 40 to 100 $\frac{2}{5} = \frac{40}{100}$	Teacher's guide p. 242	Text p. 243
	Given three numbers of a proportion, the student will find the fourth.	$\frac{\square}{3} = \frac{8}{12} \rightarrow \square = 2$ $\frac{14}{20} \cdot \frac{\square}{100} \rightarrow \square = 70$	Teacher's guide p. 249 Workbook p. 139	Text p. 250
	Given a percent, the student will express it as a decimal or a fraction.	$35\% = .35 = \frac{35}{100} = \frac{7}{20}$ $7\% = .07 = 7/100$	10 × 10 squares of graph paper Text p. 250 Worksheet Fradecent Game (*Arithmetic Teacher* 3/72)	Worksheet
	Given a whole number and a percent between 1 and 100, the student will find the percent of the given number.	6% of 50 = 3 24% of 150 = 36	Teacher's guide pp 251–2 Newspaper ads Text pp 252–3 Calculators	Text p 254 Unit Test pp 255–6 Overall evaluation: Level 7 Criterion Referenced Test

Figure 5
Curriculum Guide Format
(Sample)

Implementing the curriculum

After the curriculum has been written or revised, the words on paper must be transformed into classroom practice. This crucial transfer entails a well-planned implementation phase which should be designed as part of the curriculum development process. Major implementation considerations include:

Dissemination and familiarization. Teachers must receive copies of the curriculum and must understand what changes have been made and why. An effective procedure involves a meeting of all teachers in one or two grade levels to review the revised curriculum, preferably with a member of the curriculum committee, for a thorough discussion of the changes made, the reasons for them, and how they can be implemented. One curriculum format that has been successfully used entails placing the curriculum on 5 × 8 cards, with objectives listed on one side and suggested methods, activities, resources and evaluation procedures on the reverse side. This format allows for easy curriculum revision by replacing, deleting or adding cards.

Staff development. Curriculum revision often entails new or different objectives, revised activities or updated pedagogical techniques. Effectively implementing these changes requires workshops for some or all teachers. In addition to large-scale professional development programs, opportunities can be provided for individuals or small groups of staff members to attend conferences or workshops or visit other schools. These activities are an essential component of implementing curricular changes, particularly those suggested in this guide. Schools and school districts should seek the assistance of their own staff, teacher centers, regional educational service centers, textbook publishers and the State Department of Education for the provision of professional development programs in mathematics. Further, it is often effective to select a teacher in each school to help coordinate professional development needs and to assist other teachers in new aspects of the curriculum.

Availability of necessary instructional resources. These might include supplementary workbooks, calculators, or various manipulative materials. Nothing can be more demoralizing for teachers or more likely to defeat curriculum changes than a lack of necessary resources.

Time allocations. Sufficient time during the school day is also essential for implementing a curriculum. Given the range of skills and concepts students are expected to master and given the importance of mathematics in general, the following minimum amounts of time should be devoted to mathematics.

Time Recommendations

Grades	Minutes Per Week	Periods Per Week
1–3	225	
4–6	300	
7–12		5

It is particularly important in grades 7–12 to supplement this time through interdisciplinary study that involves mathematics so as to approach an hour per day on the average. Moreover, in many elementary schools, mathematics instruction comes just before or just after lunch. A commitment to more flexible scheduling of mathematics during a school day should be part of a district's implementation plan.

Public information program. Major curricular changes should be adequately publicized in the community. This can be accomplished through the press, radio, flyers, public meetings, or various combinations of these and other methods.

It must be kept in mind that additional support and supervision is critical throughout the initial stages of implementing a new curriculum. Special emphasis should be placed on providing necessary consultative help, meeting time and professional development opportunities for teachers and administrators.

Additional suggestions on implementing a curriculum can be found in the first publication in this series, *A Guide to Curriculum Development: Purposes, Practices and Procedures.*

5 Meeting Individual Student Needs

The mathematics curriculum developer, in trying to create and implement a sound mathematics program, must recognize the fact that extreme differences in ability and learning style exist among students. Each additional year a group of students is in school, the range of differences tends to increase.

Students differ in mental ability, level of mastery of previously taught mathematics, rate of learning, readiness for new work, interests, home backgrounds, degree of mathematical insight, attitude toward school, attitude toward mathematics, and relationship to the teacher, among other variables. It is important that teachers determine, to the degree possible, the extent of these differ-

ences early in the year and it is important that the curriculum [illegible] provide variations based upon these differences.

A mathematics program can provide for these individual differences in a variety of organizational ways:

- by grouping students from several classes for larger group presentations that utilize the individual strengths of cooperating teachers;
- by varying class sizes so that students who need greater attention can receive it;
- by scheduling differing amounts of time according to student need;
- by grouping students into teams of two or more in which the leadership role is assumed by the students under the teacher's guidance;
- by providing a special setting—a resource room or mathematics laboratory with appropriate materials—where help is available from teachers, teacher aides or talented students;
- by individualizing instruction so that students occasionally work independently (and in small groups when appropriate) at their own pace and progress along a well-planned sequence of learning experiences;
- by utilizing computer-assisted instructional materials, and
- by providing an inventory of projects and experiments for students to draw upon.

The primary vehicle by which the curriculum developer provides for individual differences is variation—variation of materials, variation of content, and variation in teaching methods.

Variety of materials

A variety of appropriate instructional materials, carefully selected and widely used, contributes to the effective teaching of mathematics and helps to provide for individual differences. For example:

- Manipulative materials such as base ten blocks, fraction bars, geoboards, etc., can be seen and handled by the student. These materials are frequently used in introductory and exploratory learning and help to provide concrete experiences.
- Representational materials such as models, pictures, diagrams, charts, graphs and tables may be used to help students make connections between concrete and symbolic representations of an idea.
- Textbooks and teachers' manuals are invaluable instructional materials and provide the basic framework for most of the mathematics to be taught.
- Workbooks provide additional and varied practice on skills and afford other opportunities for evaluating student learning.
- Programmed materials provide a structured and systematic presentation that permit students to work independently at their own pace.
- Supplementary books and pamphlets allow for individual projects and the exploration of various mathematical topics in greater depth. These

materials, used as supplements to the basic program, can help to build interest and motivation. Excellent bibliographies of such books can be found in two publications of the National Council of Teachers of Mathematics (NCTM): *The High School Mathematics Library* and *Mathematics Library—Elementary and Junior High School.*

- Audiovisual materials, including films, filmstrips, videotapes and audiotapes, can often help to introduce, summarize, or clarify various mathematical topics.
- Calculators are increasingly being used to emphasize problem-solving skills without time-consuming paper and pencil calculations, to help develop key concepts and to expedite a range of trigonometric and logarithmic calculations.
- Computers, used for enrichment, drill and practice, problem solving, recordkeeping and a range of computer-assisted instruction activities, are a growing component of a school's mathematics program.
- Community resources, including speakers and field trips, can be used to develop an awareness of the importance and utility of mathematics in our society.

Effective curriculum development makes reference to available materials for specific objectives and encourages the use of a variety of these materials.

Variety of content

Although there is a basic core of mathematical content which all students should be expected to master, differentiation in content should be incorporated into a curriculum to meet varying levels of student ability, need and interest. For example:

- A well defined enrichment strand should be developed containing challenging or difficult topics for faster learners.
- A supplementary remedial program that provides alternative opportunities to master skills and concepts needed for adult mathematical literacy should be available for slower learners.
- Infrequently used or difficult topics should be deleted for slower learners.
- Recreational activities, including puzzles and games, should be added for all students.
- Skills and concepts ordinarily taught in the next grade might be considered for gifted learners.
- Some topics might be delayed until later in the year or moved to the next grade for slower learners.

More specifically, content and methods for faster and gifted learners might include:

- more independent use of the textbook;
- use of more challenging worksheets and nonroutine problems;
- use of varied mental arithmetic shortcuts;
- use of a more challenging textbook;
- reading of supplementary books and pamphlets;
- use of recreational mathematics exercises;

- creation of their own materials and problems;
- use of more challenging tests, and
- independent mathematics and interdisciplinary projects.

Similary, content and methods for slower learners might include:

- greater attention to concrete and pictorial materials and experiences;
- longer and more frequent use of manipulative and visual aids to supplement textbooks;
- use of textbooks with appropriate readability levels;
- closely supervised reading, study and discussion of printed material;
- slower introduction of successive steps in a process;
- frequent reteaching of meanings and procedures;
- frequent reviews;
- shorter listening and practice periods;
- frequent dramatization and diagramming of problem situations;
- use of motivational games and enrichment activities;
- opportunities for small group work;
- short, but frequent, diagnostic testing;
- more practical mathematics applications and situations, and
- use of calculators for routine computation.

Overall, regardless of the range of individual student differences, the mathematics curriculum developer can help to assure that the needs of individual students will be best met through a program that:

- specifies objectives in terms of student behaviors;
- selects and provides materials and learning experiences that correspond with the objectives;
- provides for the diagnosis of each student's mathematical strengths and weaknesses and degree of skill and concept mastery;
- provides prescriptions and programs that address these weaknesses and lead to mastery;
- provides a careful introduction to each basic mathematical topic;
- incorporates the use of concrete and pictorial materials;
- provides sufficient practice on each skill;
- incorporates a recordkeeping procedure to monitor individual student progress, and
- systematically evaluates student, school and district progress.

6 Student and Program Evaluation

The monitoring of student progress and the evaluation of a mathematics program are ongoing processes. They are indispensable to a sound program and should be built into a mathematics curriculum as it is developed. Although evaluation is often limited to an assessment of outcomes in terms of student learning and student behavior, it should also involve the continual appraisal of the instructional process that leads to these outcomes as well as the factors that directly affect student outcomes. Such factors as staff, facilities, equipment, administrative organization, school and classroom procedures, and course content all contribute to the effectiveness of the program and to students' ultimate mathematical achievement.

Within this context, the purpose of evaluation is to enable teachers, administrators, parents, students and members of the community to determine whether the goals and objectives of the program are being realized and whether further changes are needed. Thus an effective evaluation program:

- provides important information on the needs, accomplishments and effectiveness of the mathematics program;
- helps locate programmatic strengths and weaknesses;
- is used to build a stronger curriculum by providing a basis for appropriate changes in objectives, learning activities, instructional methods and materials;
- helps allocate resources to areas of greatest need;
- assesses student achievement in order to place students in appropriate instructional programs;
- assesses student achievement at the end of curricular segments in order to specify what has been learned and to determine readiness for further instruction, and
- monitors individual student progress in an ongoing manner.

Program evaluation

The most common method of program evaluation is student testing. Tests of a wide variety provide a wealth of information about groups of students as well as individual students. The type of test chosen depends upon the information desired and the use to which it will be put. The effective evaluation of a mathematics program usually entails a balanced combination of norm-referenced tests and criterion-referenced tests.

Norm-referenced or standardized tests. The majority of these tests include mathematics computation, concepts and application components. These tests are usually well-written and have high reliability and statistical validity. However, the decision to use a particular norm-referenced test should be made carefully on the basis of its curricular validity, that is, its consistency with the specific goals and objectives of a district's instructional program. Since norm-referenced test items measure only a portion of a mathematics curriculum and since items are selected in such a way as to eliminate those that are too easy or too difficult for students in a large sample population, analysis and interpretation of test results must be made within the context of a local district's objectives and the sequencing of these objectives.

Norm-referenced tests can help to:

- provide a general comparison of an individual or a group with those in a larger school, district, state, regional or national population;
- indicate a general achievement level of students;
- indicate broad areas of comparative strength and weakness in the curriculum;
- provide an initial screening of students doing significantly better or poorer than the mean, and
- provide a rough basis for grouping students for instruction.

Criterion-referenced tests. Tests of this type are based upon a known and specified set of criteria usually stated in the form of objectives. These criteria describe performance that is to be used as evidence that a particular concept has been learned or a particular skill mastered. For example, "Given five exercises involving the subtraction of 1- and 2-place decimals, the student will correctly answer at least four problems." Rather than comparing a student's achievement with those of other students, as in norm-referenced testing, criterion-referenced tests measure each student's performance against a predetermined standard. Most significant, in terms of evaluating a district's mathematics program, is the opportunity to design criterion-referenced instruments having a high correlation with curricular objectives. Thus, criterion-referenced tests can help to:

- give specific information as to what tasks students can and cannot do;
- provide information that indicates whether instruction or reinstruction is needed;
- provide information, at the objective level, that is directly responsive to local needs, and
- provide valuable information to help monitor individual student progress.

It is important to note that tests used to measure a student's achievement, whether norm-or criterion-referenced, are not appropriate for use in diagnosing students' specific strengths and weaknesses. Achievement tests will identify only *general* areas of student difficulty; diagnostic tests should then be used to identify *specific* troublesome areas. However, carefully constructed criterion-referenced tests can be effective instruments for conducting a preliminary diagnosis of student difficulty.

In addition to tests, important information for purposes of program evaluation can be attained from *interviews, surveys and classroom observations*. Interviews and surveys can elicit valuable data on perceptions of programmatic strengths, weaknesses and needs, preferences for texts and other materials, degree of attention to various topics and a range of related topics. In short, periodic interviewing and/or surveying helps provide information on student, teacher, administrator or parent perception of which parts of the mathematics program are working and which are not.

Similarly, classroom observations provide an opportunity to assess the translation of a paper curriculum into classroom practices. Observations can help determine the degree of implementation of various curriculum objectives, student response to curriculum activities and additional data on techniques and methods used by teachers. Classroom observations are also valuable means to evaluate curricular objectives from the affective and psychomotor domains.

Student monitoring

Given the sequential nature of mathematics programs, where most skills depend upon the mastery of previous skills, it is extremely important to monitor student progress on a skill-by-skill basis and provide a means for this information to be passed from teacher to teacher and from school to school. Such monitoring of the mastery and subsequent retention of skills can be accomplished through a combination of 1) teacher-developed quizzes and tests, 2) district or textbook devised unit tests, 3) school or district-wide criterion-referenced tests, 4) analysis of homework and other assignments or 5) observation of students during class. In addition, as a means of assessing readiness, or for a more detailed determination of individual strengths and weaknesses, a range of diagnostic tests can be used.

Whatever method is used, student achievement should be recorded in a manner which readily provides a summary of each student's mathematical progress. A common practice is to develop a mathematical skills record card that is matched directly to the curriculum and to the assessment instruments in use. Such a record card should be updated at least quarterly and provide a means to display exposure to, mastery and retention of key curricular objectives. A record card is particularly useful for transferring valuable information about students as they are promoted from grade to grade and from school to school. When record cards are used in conjunction with spring and fall criterion-referenced assessment, teachers can readily determine which skills have not been mastered or have been lost during the summer and must be retaught. Samples of cards are provided in Appendix E.

Guidelines for constructing effective mathematics tests that help to monitor student progress include these features:

- items should be related directly to the objectives of instruction;
- the type of questions should be determined by the objectives being measured (e.g., short answer for some objectives, detailed proof, essay or construction for others);
- items should be clearly written in concise, unambiguous, grammatically correct language so that students understand exactly what is wanted;
- "trick questions" should be avoided, except perhaps for extra credit;
- items should be included which test students' ability to apply ideas learned rather than merely regurgitate facts (assuming that prior instruction included such application);
- the test should be short enough to provide adequate time for completion, unless it is understood by students to be a speed test;
- items should usually be arranged in ascending order of difficulty;
- items should be evaluated after use, with some stored for future use and others discarded or rewritten, and
- the test should be taken by the teacher or tried out on others prior to its administration to check on time, clarity and appropriateness.

Part II
Mathematics
Curriculum Content

The K–8 Program 7

An effective kindergarten through 8th grade mathematics curriculum interrelates the learning of concepts, skills and applications with the developmental stages of children. Instructional activities should be presented on three levels: concrete, representational and abstract. The understanding of concepts, built on experiences with concrete materials and representations of materials, forms the basis for meaningful skill development. Mastered skills are applied through problem solving, the principal reason for the study of mathematics.

This interdependence is clear when the concept of number expands to the concept of place value which underlies the mastery of addition and subtraction algorithms—all three built upon concrete and representational experiences. This in turn allows for an application, such as keeping a running balance in a checkbook. Similarly, the concept of ratio has direct bearing on the calculation of percent, which in turn allows for a determination of discount.

A major goal of the K–8 mathematics curriculum developer should be a balanced treatment of all three elements—concepts, skills and applications. Too often, concepts and applications are subordinated and the major part of the curriculum becomes the isolated "coverage" of computational skills. While certainly essential, these computational skills must arise out of a firm conceptual base and must then be used within the context of problems and applications.

The suggestions made in this section, and embodied in the suggested scope and sequence that follows, address a range of issues that have often been ignored while developing K–8 mathematics curricula. By adopting a particular textbook series and calling it the curriculum, schools abdicate their responsibility to develop a comprehensive mathematics program designed to meet their particular needs and the diverse needs of their students. The assumption of this responsibility entails the careful consideration of a developmentally sound sequence of objectives, the importance of concrete and pictorial learning experiences, a strong emphasis on problem solving, a realistic approach to measurement, the unique problems of 7th and 8th grade, and the emerging role of calculators and computers. These issues are discussed below.

Sequencing

Too often what we know about how children learn mathematics seems to have little to do with how we and many of our textbooks sequence the elementary mathematics curriculum. For example, mastery of basic addition and subtraction facts cannot be expected before children have had numerous concrete experiences with numbers. Yet we frequently expect children to begin learning addition combinations before they have fully developed the concept of number. For many children, full mastery of addition and subtraction facts is not possible until age eight or 3rd grade.

Similarly, few children have mastered the concepts and skills required for *understanding* regrouping until around age eight. However, *mastery* of two-place addition and subtraction with regrouping is often expected as early as 2nd grade. Applying the regrouping concept in addition and subtraction problems requires the concept of number, the ability to count on from tens, a mature understanding of place value, mastery of addition and subtraction facts, and the ability to handle two or more steps at one time. Thus, **for average students**, the full understanding and retention of addition and subtraction with regrouping comes approximately at the end of 3rd grade.

In *The Piaget Primer* Labinowicz notes:

> At a time when most children are just grasping the concept of number, the scope and sequence chart requires that they tackle exercises in addition and subtraction that require an understanding of place value concepts. Usually this expectation includes dealing with problems (like 27 + 19) at an abstract, symbolic level with only a short exposure at a pictorial level, and with no experience with related materials. The children are given little opportunity to construct their place value relations before applying them. Those who begin to develop their own methods find that the teacher's method cannot be superimposed on theirs; consequently, they give in to authority and memorize and follow blindly a set of rules.[2]

Similar developmental readiness issues arise around fractions, decimals, long division, measurement conversions and many other topics. Understanding grows slowly, suggesting the need for careful examination of the traditional K–8 mathematics sequence found in most textbooks. A suggested developmentally based scope and sequence that attempts to address these issues appears in Tables 4 through 7.

Concrete and representational experiences

> The depressing thing about arithmetic badly taught is that it destroys a child's intellect, and to some extent, his integrity. Before they are taught arithmetic, children will not give their assent to utter nonsense; afterwards they will. Instead of looking at things and thinking about them, they will made wild guesses in the hope of pleasing a teacher or an examiner.[3]
>
> —W.W. Sawyer

The heart of an effective, well-taught mathematics program is the careful development of concepts and skills through extensive concrete and pictorial experiences and activities. Manipulating concrete objects and then constructing pictorial representations are essential elements of a program wherein mathematics is performed with understanding rather than simply remembered and regurgitated.

Again, the suggested scope and sequence in Tables 4 through 7 incorporates a sequence for most skills that builds from concrete experiences to representational experiences to abstract level mastery. Examples of materials that can assist teachers in the inclusion of concrete and representational experiences in their classes are:

bean sticks and/or bundles of sticks
multi-base or base ten blocks, for example, Number Blox, Dienes Blocks
trading materials such as colored chips
attribute materials, for example, Attribute Blocks, People Pieces
patterning materials, for example, Pattern Blocks, colored cubes
Cuisenaire rods
geoboards
tangrams
fraction materials, for example, fraction bars, fraction tiles
measuring devices including rulers, trundle wheels, pan balances, scales, bottles, jars, graduated cylinders, thermometers, etc.
graph paper, colored construction paper

Although available commercially, many of these materials can be made by students or teachers for classroom use.

Problem solving

In *An Agenda for Action* the National Council of Teachers of Mathematics recommends that "problem solving be the focus of school mathematics in the 1980s" and that "basic skills in mathematics be defined to encompass more than computational facility."[4] If problem solving is to become the fundamental theme of the mathematics curriculum then all topics must be presented in the context of problem solving, i.e., questioning, reasoning, finding patterns, organizing information, making predictions, using models, etc.

Such an approach means that basic computational skills should be presented through an active involvement with materials and situations so that problem-solving skills can develop. For example, multiplication and division would begin with problems that arise from manipulating equivalent sets, constructing arrays and exploring patterns of multiples.

Problem solving, as outlined in Tables 4 through 7, also means an emphasis on topics such as classification, patterning, ordering, graphing, probability and statistics, estimation, number sentences and social applications of computational skills. Thus, problem solving is enlarged well beyond the traditional limits of "guess the operation" type story problems.

For example, patterning entails the reproduction, recognition and extension of various geometric and number patterns of increasing sophistication that develop a foundation for more advanced inductive and deductive reasoning skills. Similarly, estimation, often relegated to rounding and computing, entails organizing information, making judgments and then making predictions—all problem-solving skills. In turn, good estimation skills allow students to judge the reasonableness of answers to problems.

Overall, the curriculum developer must incorporate appropriate problems to illustrate and complement mathematics skills and concepts.

It is important to note that, unlike computation and measurement skills, problem-solving content is difficult to delineate in detail because, at its most instructive level, problem solving entails the nonroutine and open-ended exploration of a problem. That is, it requires a higher degree of informality and a less rigidly defined structure as students are given the opportunity to explore processes that vary from problem to problem and from student to student. Effective instruction in mathematics problem solving requires an atmosphere in which openness, communication, cooperation and creativity can occur. Teachers can set an example by actually solving problems and using student ideas.

Thus, the interweaving of problem solving into all areas of the mathematics curriculum requires flexibility, creativity, and a willingness and encouragement to supplement presently available textbooks.

Measurement

The measurement strand of the K–8 curriculum provides numerous opportunities to reinforce other parts of the mathematics curriculum. Units can be counted and objects compared; the numbers expressing weights, lengths, capacities, areas and volumes can be added, subtracted, multiplied and divided. Many measurement activities provide concrete experience with fractions, build an understanding of the nature of operations, and encourage estimation. Further, much of the problem solving encountered in real life (how much? how heavy? how long? how big?) concerns measurement.

As with all mathematical understandings, children's understanding of measurement develops slowly. Many experiences are needed with concrete, nonstandard units before standard measures in the metric and customary systems can be used. For each area of measurement, the effective curriculum builds from informal play and exploratory experiences to direct comparison and ordering, to use of nonstandard units, and finally to use of standard units. Estimation and problem-solving skills should be applied throughout the sequence.

An unfortunate reality of today's measurement curriculum is the need to address both the metric and customary systems in a balanced fashion. Despite surface appearances and albeit slowly, the United States is unquestionably in the process of converting to the metric system. Business and industry are not generally advertising this fact, but they are quietly converting and a knowledge of metrics will be a requirement for the world of work that today's students will face.

Given the slow rate of change, we cannot teach meters and kilograms to the exclusion of yards and pounds. Students still require a full working knowledge of the customary system and, during this transition period, the effective curriculum must balance the teaching of both systems.

7th and 8th grade considerations

The 7th and 8th grade mathematics program is often an isolated island between the K–6 and 9–12 programs. To assure a clear articulation of skills and concepts through 8th grade, this guide addresses the K–8 program as a single entity. However, in planning an appropriate middle or junior high school program, mathematics curriculum developers should address several issues unique to these grade levels.

- The middle or junior high school curriculum must be a logical and coordinated extension of the K–5 or K–6 program. Record cards employed in earlier grades must be extended and maintained through at least 8th grade.
- Consideration must be given to the broad range of student abilities. Some students will have completed nearly all of the K–8 objectives by the end of 7th grade and will be ready to take algebra I in 8th grade. Other students will still be mastering objectives commonly found in 4th and 5th grade and will still require extensive experience at the concrete and pictorial levels.
- Many students, having mastered whole number, fraction and decimal computation, are ready for much greater attention to the application of these skills and other problem-solving activities. For these students, this is an appropriate time to increase use of the calculator for routine computation, freeing valuable time for focusing on specific problem-solving skills.
- The skills and concepts of percents must receive sufficient attention during 7th and 8th grade. Standardized tests reveal serious weakness in this area and curriculum developers must provide an appropriate sequence and sufficient time for mastery of these skills. Further, for the majority of students who proceed to algebra I, 8th grade often provides the last formal instruction on these essential skills and understandings.
- Finally, the 8th grade program should provide opportunities for more advanced students to proceed beyond the topics listed in the suggested scope and sequence in Tables 4 through 7. In addition to providing algebra I for the most able students, a range of pre-algebra topics including investigation of radicals and the real number system, and exposure to equations, variables, algebraic expressions and logic may be added for some other students.

Calculators

The widespread availability of calculators cannot be ignored when developing a mathematics curriculum.

The inexpensive hand-held calculator dramatically alters the need for mastery of complex computation that often occupies months and even years of

teaching. The suggested scope and sequence in Tables 4 through 7 goes no further than the addition and subtraction of 4-digit numbers, the multiplication of 3-digit by 2-digit numbers and the division of 3- and 4-digit numbers by 2-digit numbers. The algorithmic skills required to attain mastery at these levels are deemed sufficient and the time required to master more complex computation can only come at the expense of other more important skills and concepts. One of the strongest arguments for calculator use is the opportunity to increase significantly student exposure to real world problems and applications.

In *A Resource for Problem Solving* the Ohio Department of Education recommends that a calculator should be used when it:

- extends personal capability to do problem solving;
- speeds up a search for patterns;
- enables more examples to be considered;
- removes the anxiety of computation failure;
- permits attention to be directed to other problem aspects;
- provides motivation and confidence that a problem can be solved;
- offers access to mathematical ideas not available without it;
- eliminates tedious or routine computations;
- allows problems of greater intricacy to be attempted;
- contributes to further applications and explorations of related topics, and
- saves time within the curriculum without significantly detracting from other objectives.

The Ohio recommendations[5] also note that calculators should not be used:

- if there is no advantage to performing the learning activity with a calculator;
- so extensively that opportunities to use and maintain personal computational skills are significantly diminished;
- in competitive situations in which all students do not have equal access to a calculator, and
- if the situation is artificially contrived for the express purpose of using a calculator and has no other educational outcomes (excepting, however, certain introductory activities to teach students how to use the calculator).

Classroom sets of hand-held calculators should be available for student use in all schools and their use integrated into the mathematics curriculum.

Computers

Each passing year brings greater availability of computers to elementary, middle and junior high schools. The relatively low cost of microcomputers has accelerated this trend and brought the enormous potential of computers directly into the mathematics curriculum.

There are several ways in which computers can interact with and influence the mathematics curriculum.

The computer and the development of computer literacy itself can be a curricular topic. While programming in BASIC is included as a topic in Table 6,

this skill is an application of many of the topics addressed in the problem-solving strand. Among them are recognizing contradictions and inconsistencies, comparing predictions to outcomes, formulating generalizations, drawing conclusions, revising estimates and constructing flow-charts.

In addition to interacting with computers and understanding programming, computer literacy should also include a knowledge of computer components, awareness of the impact and limitations of computers, and a positive attitude toward computers. Also, computers can be used effectively as tools of instruction. The growing availability of mathematics software allows computers to be used for drill and practice, computer-assisted instruction, simulation, games and problem solving. These computer resources provide instant nonjudgmental feedback to students and create environments that can enrich a mathematics program.

The growing importance of computers in the world of work makes it clear that knowledge and competency in this area is a necessity. In a high technology state like Connecticut, every student must have formal training in, and hands-on experience with, computers.

K–8 scope and sequence

The suggested K–8 mathematics program scope and sequence outlined in Tables 4 through 7 attempts to incorporate the issues and considerations discussed previously. It begins with a developmental framework (Table 4) that identifies various characteristics and diagnostic tasks for each age/grade level. The K–8 program is then presented in the following strands and substrands:

Number work/operations (see Table 5)

- counting
- numeration/place value
- addition/subtraction
- multiplication/division
- fractions
- decimals
- ratio/proportion, percent
- miscellaneous

Problem solving (see Table 6)

- classification
- ordering (seriation)
- patterning
- graphing
- estimation
- probability and statistics
- number sentences, word problems, applications
- miscellaneous

Measurement/geometry (see Table 7)

- time
- money
- measurement
- geometry

The tables demonstrate how skills and concepts within a substrand are developed over time (read horizontally), and how skills and concepts within an age or grade are interrelated and reinforced (read vertically). Moreover, when

visualized as one large chart, the tables provide a systematic organization of a K–8 mathematics program.

Although the tables are organized by age and grade level, they should be viewed as a continuum of skills and concepts. The use of two grade headings, (e.g., 3–4 or 5–6) is meant to reinforce the fact that while some students should be ready to learn a skill at the earlier grade, other students should not be expected to attain mastery until the later grade. **It is recognized that the suggested scope and sequence pertains most directly to the "average" student and that both the degree of depth and the age/grade placement of topics will be altered to meet individual needs.**

The adoption and/or development of such a scope and sequence is important for exercising local control over the mathematics curriculum. Too often this control arises from textbooks and from standardized tests that rarely meet local needs. For an effective mathematics program, one first establishes a workable scope and sequence based upon what we know about children's cognitive development, and *then* adapts available texts and tests to this curriculum, rather than the reverse. The curriculum developer will find that no one text is complete and supplementary materials will be needed. This process is particularly important given the attention to concrete and pictorial experiences, to problem solving and to reordering the traditional sequence of skills and concepts that can be found in the following tables.

It should also be noted that while the sequencing of certain skills and concepts has been altered from what has traditionally been done, the overall scope of the K–8 program has not been reduced in the areas of number work/operations and measurement and it has been expanded in the area of problem solving. Moreover, the sequence does deviate in some respects from standardized tests commonly used in grades 2–5. This, however, is viewed as a weakness on the part of the tests, not of the suggested curriculum. In fact, it is believed that this reordering can be a preventative measure that diminishes the number of students who are unsuccessful in mathematics.

SUGGESTED K–8 SCOPE AND SEQUENCE

		Pages
Table 4	Developmental Framework	46–47
Table 5	Number/Work Operations	48–53
Table 6	Problem Solving	54–61
Table 7	Measurement/Geometry	62–65

Table 4
Developmental Framework

Grade	K–1	1–2
Age	**5–6 years**	**6–7 years**
DEVELOPMENTAL STAGE	Preoperational (Concrete Operations) Pre-Number	Preoperational (Concrete Operations) Pre-Number/Number
CHARACTERISTICS OF THE STAGE	Easily swayed by distractions. Distracted from correct solutions in problem situations by background color, shape of a box, arrangements. Cannot hold more than one idea or variable in mind at one time. Parts are not related to the whole. Not aware of contradictions.	Less easily distracted by multiple ideas and contradictions. Able to classify objects, count objects with an appreciation of number and order objects according to size. Conservation of number.
DIAGNOSTIC TASKS	Counting with objects (can count five blocks). Puts objects in one-to-one correspondence (matches 5 cups with 5 saucers) More/less (can pair objects from two sets to determine which set has more/less).	Counting on from 3, 5, 10 (Given 5 pennies and then 3 more, can count on from the 5–6, 7, 8) Conservation or concept of number, i.e., number remains the same regardless of arrangement (Given □ □ □ □ □ / □ □ □ □ □, understands that both rows have the same number) Specific number more/less (Given two sets of objects, can tell which one has more/less and how many more/less)

Grade	4–5	5–6
Age	**9–10 years**	**10–11 years**
DEVELOPMENTAL STAGE	Concrete Operations Class Inclusion	Concrete Operations (Formal Operations)
CHARACTERISTICS OF THE STAGE	Classification skills with physical objects—ability to construct hierarchies. Conservation of weight.	Begins to develop abstracting skills. Still needs physical materials to learn concepts and to solve problems. Does not recognize ratio relationships.
DIAGNOSTIC TASKS	Conservation of weight (understands that the weight of a clay ball is unaffected when its shape is changed).	

2–3	3–4
7–8 years	**8–9 years**
Preoperational Concrete Operations	(Preoperational) Concrete Operations
Pre-Number/Number	Class Inclusion
Can place objects in 2 overlapping classes and justify choice. Can construct an ordered series. Can order by size and color. Understands relationships of parts to whole—whole is equal to the sum of its parts. Conservation of length, liquid amount.	Class inclusion concept—all robins are birds, but not all birds are robins. Mentally able to hold parts and whole to make comparisons. Still needs physical materials to solve ordering problems. Conservation of solid amount, area. Transitivity: If A is smaller than B and B is smaller than C, then A is smaller than C
Parts/Whole (understands that a set of eight objects "is the same" as a set of five objects and a set of three objects). Conservation of length (understands that 5 stick "road" like _ _ _ _ _ is the same length as one like \/\/\ ). Conservation of liquid amount. (understands that the amount of liquid in a container remains the same when poured into another container despite different liquid heights).	Conservation of solid amount (understands that the amount of clay in a ball is unaffected by changing its shape or by subdividing it. Conservation of area (understands that the area covered by a number of two dimensional objects is unaffected by their arrangement).

6–7	7–8
11–12 years	**12–13 years**
Concrete Operations Formal Operations	(Concrete Operations) Formal Operations
Able to draw abstractions from other abstractions. Recognizes the ratio relationship.	Concept of displaced volume. Complex classifications—creates classification systems.
	Conservation of displaced volume (understands that the volume displaced by an object is dependent on the volume of the object and is independent of weights or shape).

Table 5
Number Work/Operations

Grade	K–1	1–2
Age	**5 to 6 years**	**6 to 7 years**
COUNTING	Rote counting to 10, 20, 30 One-to-one correspondence Counting with objects to 5, 8, 10 . . . Comparing sets: same, more, less	Comparing sets: one, two . . . more one, two . . . less Rote counting by tens Counting on from 3, 5, 10 Counting back from 5 to 3, 8 to 5 Reverse counting from 10 to 0, 20 to 0 Counting on with models from 10 to 19; 10, 20 to 29 10, 20, 30 to 39
NUMERATION/ PLACE VALUE	Match number symbols with sets of objects or pictures of objects to 5, 10	Read and write numbers to 5, to 10 Read and write numbers from 11 to 19 Represent the numbers with concrete materials Write the numbers in expanded notation from a picture model
ADDITION/ SUBTRACTION	Addition at the counting level with objects or pictures of objects	Explore addition combinations with concrete materials Record combinations at the representational level Addition at the concept level (conservation of numbers) Subtraction with objects ("Take-away" problems, "how many more/less than" problems with objects and pictures) + and = signs
MULTIPLICATION/ DIVISION		Explore multiplication as repeated addition with concrete materials and record as pictures

2–3	3–4
7–8 years	**8–9 years**
Counting on by ones from 90 to 120 Counting on by tens to 130 . . . 900 and beyond Counting on by tens from 7 to 137 and beyond Skip counting by twos, fives, threes as multiples: 2, 4, 6 . . . as addends: 3, 8, 13 . . . Count sets of objects by grouping by twos, fives, or tens	Counting on by tens to 1010 and beyond Counting on by hundreds to 1200 and beyond
Construct numbers to 100 using concrete materials Read, write and compare numbers to 100 Construct and rewrite 2-digit, then 3-digit numbers for regrouping (36 = 20 + 16) Mastery of tens and ones in two-digit numbers (2 tens + 6 ones = 20 + 6 = 26) Add/subtract a 10 from numbers to 130 Write number to 999 in expanded notation from picture model	Construct read, write and compare numbers to 9999 Construct and rewrite three-digit and four-digit numbers for regrouping (236 = 200 + 20 + 16 = 100 + 120 + 16) Mastery of tens in three-digit numbers (120 = 12 tens) Inequality signs >, < Add/subtract 100 from numbers to 990 Rounding to nearest 10, 100
Subtraction at the concept level − sign Add/subtract as fact families Mastery of basic addition and subtraction facts to 10 Concrete and representational experience with addition and subtraction facts, 11 to 18 Addition of doubles to 18 Addition of one more than a double 2-digit addition and subtraction—no regrouping Terms "addend" and "sum" (See estimation strand.)	Mastery of addition and subtraction facts to 18 Introduction of regrouping for add/subt. with 2-digit, then 3-digit numbers (See estimation strand.)
Multiplication as repeated addition × sign Explore fact combinations with concrete materials and record as pictures Introduce mult./div. as an array and symbolically as a mult./div. frame	Mastery of multiples of 2, 3, 4, 5 Division by factors, 2, 3, 5 Multiplication and division as fact fact families (3 × 5 = 15 5 × 3 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3) Terms "Factor" and "Product"

Table 5 (continued)
Number Work/Operations

Grade	K–1	1–2
Age	**5–6 years**	**6–7 years**
FRACTIONS	Concrete experiences for verbalizing meaning of one-half Use sheets of paper, cakes, pies, apples, etc. for dividing into 2, 3 and 4 equal parts	Concrete experiences for verbalizing meaning of one-half, one-third and one-fourth Use sheets of paper, cakes, pies, apples, etc. for dividing into 2, 3, and 4 equal parts
DECIMALS		

Grade	4–5	5–6
Age	**9–10 years**	**10–11 years**
COUNTING		
NUMERATION/ PLACE VALUE	Construct, read, write and compare numbers to hundred thousands Rounding to nearest 10, 100, 1,000 10s and 100s in 4-digit multiples of 100 (1,200 = 12 hundreds = 120 tens)	Construct, read, write and compare numbers to millions Rounding numbers
ADDITION/ SUBTRACTION	Mastery of addition/subtraction of 2- and 3-digit numbers with regrouping Column addition	Reinforcement of addition/subtraction of 2-, 3- and 4-digit numbers to 10,000
MULTIPLICATION/ DIVISION	Mastery of multiplication/division basic facts Multiplication of 2- and 3-digit numbers by a one-digit number Multiplication of multiples of 10 by 10 and 100 (80 × 10; 50 × 100) Division of 2- and 3-digit numbers by a one-digit number Division with remainders as fractions Mental Computation (43 × 6 = 240 + 18 = 258) Prime numbers to 100	Multiplication/division of 2- and 3-digit numbers by 1- and 2-digit numbers Multiplication/division by 10, 100, 1,000 Divisibility rules for 2, 3, 5, 10 Prime factorization of numbers to 100 Least Common Multiples (LCM of 8 and 12) Greatest Common Factors (GCF of 32, 40)

2–3	3–4
7–8 years	**8–9 years**
Fraction symbol for one-half (½) Explore parts to whole relationship with two, three and four equivalent subsets Concrete and pictorial experience labeling parts of a whole, shading pictures and naming pictures for ½, ⅓, ¼, ⅕, and ⅙ Name pictures representing ⅔, ⅚, ¾ etc. Name whole using objects and pictures as ¾, ⅞, $^{10}/_{10}$	Fraction symbols for halves, thirds, fourths and eighths Label parts of a whole and shade pictures representing halves, thirds, fourths, fifths, sixths, and eighths Fraction names for one Addition sentences for picture models ($\square^{\square}_{\square}$ <=> ½ + ¼ + ⅛ + ⅛ = 1) Equivalent fractions using picture models ½ = ¾ Fractional parts of whole number sets using concrete and picture models ⅓ of 9, ⅔ of 6)
	Decimal notation for money ($3.24), pennies and dimes place

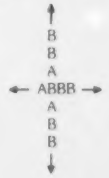
6–7	7–8
11–12 years	**12–13 years**
Exponents ($5^3 = 5 \times 5 \times 5 = 125$) Expanded notation with exponents ($1463 = 1 \times 10^3 + 4 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$)	Scientific Notation ($1463 = 1.463 \times 10^3$)
Commutative and associative properties of addition	Closure Additive identity
Commutative and associative properties of multiplication Distributive property of multiplication over addition and subtraction Reinforcement of multiplication of 3-digit numbers by 2-digit numbers Division of 3- and 4-digit numbers by 2-digit numbers Division with remainders expressed as fractions and decimals Reinforcement of prime factorization, least common multiples and greatest common factors.	Closure Multiplicative identity

Table 5 (continued)
Number Work/Operations

Grade	4–5	5–6
Age	9–10 years	10–11 years
FRACTIONS	Terms numerator and denominator Addition of fractions with like denominators Renaming of fractions as equivalents ($\frac{1}{4} = \frac{2}{8}$, $\frac{2}{3} = \frac{4}{6}$) Addition of fractions with unlike denominators, renaming one in terms of the other ($\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$) Fractional parts of whole numbers ($\frac{3}{4}$ of 24 and $\frac{3}{4} \times 24$) Whole number division with remainders expressed as fractions Ordering fractions with like denominators	Conversion of mixed numbers and improper fractions with models Equivalent fractions by multiplying by one as a fraction ($\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$) Reduce fractions to lowest terms Addition/subtraction of fractions converting one denominator, 2 different denominators, mixed numbers Multiplication of fractions with picture models Multiplication of fractions, mixed numbers Division of whole numbers by fractions and fractions by whole numbers at the concrete and pictorial level Ordering fractions with like and unlike denominators
DECIMALS	Addition/subtraction using money notation Write fractions/decimals from picture models of tenths, hundredths Read and write numbers to 2 decimal places as an extension of place value system Addition/subtraction with regrouping of 1- and 2-place decimal numbers	Ordering 1- and 2-place decimals Read and write numbers to 3 decimal places Rounding to nearest tenth Rename decimals as fractions and their equivalents (.72 = 72/100 = 720/1000 = .720) Rename fractions with denominators of 10, 100, 1,000 as decimals Addition/subtraction of 2- and 3-place decimal numbers
RATIO, PROPORTION, PERCENT		Identify partially shaded regions of 100 parts or partitioned sets of 100 objects as a decimal and percent Convert 2-place decimal numbers and fractions with denominator of 100 as percents Convert fractions with denominators of 2, 4, 5, 10, 20, 25, and 50 to fractions with denominators of 100 and rename as a percent
MISCELLANEOUS	Concept of negative number using temperature, losses, etc. Integers (−20 to 20) on a number line Introduce calculator to reinforce place value concepts	Square numbers to 20 Integers (−100 to 100) on a number line

6–7	7–8
11–12 years	**12–13 years**
Mastery of addition and subtraction of fractions and mixed numbers Mastery of multiplication of fractions and mixed numbers Concept of reciprocal Division of fractions by whole numbers and whole numbers by fractions Experience with fractions divided by fractions Ordering fractions	Reinforcement of addition, subtraction and multiplication of fractions and mixed numbers Mastery of division of – fractions by whole numbers – whole numbers by fractions – fractions by fractions
Reinforcement of addition/subtraction of 2- and 3-place decimals (horizontal and vertical form). Multiplication of decimals using fractional equivalents ($.3 \times .06 = 3/10 \times 6/100 = 18/1000 = .018$) Multiplication of 1- and 2- place decimals Ordering 1-, 2- and 3-place decimals Division of decimal numbers by whole nos. Division of whole numbers by decimals using fractional equivalents ($32 \div .4 = 32 \div 4/10 = 32 \times 10/4 = 80$) Experience with division of 2 decimal nos. Mul./Div. of numbers by 01, .01, 0001	Mastery of multiplication of 1-, 2- and 3-place decimals by 1- and 2-place decimals Mastery of division of 1- and 2-place decimals by 1- and 2-place decimals Fractional percent gain or loss
Express the comparison of two sets as a ratio Name equivalent ratios Equivalent ratios expressed as a proportion Find missing number in a proportion $\left(\frac{3}{10} = \frac{9}{\square}\right)$ Rename percents as fractions and decimals Use proportions, fractions and/or decimals to find the percent of a number	Use proportions to find a) the percent of a number b) what percent one number is of another c) the number of which a given number is a given percent Find percent gain or loss
Ordering integers Addition/subtraction of integers Rename fractions, decimals or percents as equivalents ($4/5 = .8 = 80\%$, $.85 = 85/100 = 85\%$, $40\% = 40/100 = 2/5 = .4$)	Square roots of perfect squares to 144 Use a calculator to approximate square roots Multiplication/Division of integers Rename fractions, decimals or percents as equivalents Ordering rational numbers expressed as fractions and decimals Additive and multiplicative inverses Inverse operations

Table 6
Problem Solving

GRADE	K–1	1–2
AGE	5–6 years	6–7 years
CLASSIFICATION	Use materials such as buttons, shells, stones, to freely explore groupings. Sort by one attribute such as color, shape, or size, e.g., sorts a set with 2 colors into 2 sets. Use 3, 4, colors (shapes, sizes).	Identify an object by referring to each of its attributes, e.g., show student an object such as a People Piece and student will be able to identify the piece by its sex, girth, height and color of clothes. Classify objects by referring to any of their attributes, e.g., given a set of objects such as Attribute Blocks, student will classify by color, shape or size or by two attributes such as color and shape. Find missing objects on a matrix such as the one below.
ORDERING (SERIATION)	Simple ordering by size, height, weight Order numbers 0 to 10	Construct an ordered series of up to 8 objects based on one attribute (arrange books from thinnest to thickest) Order numbers to 19 Ordinal numbers (first, second, . . . tenth)
PATTERNING	Reproduce AB patterns with blocks, shapes Extend AB patterns with blocks, shapes Reproduce and extend AB patterns involving shapes, colors or sizes or paper	Extend more complex patterns (AABAAB, ABABBABBB) Extend patterns both forward and backward, horizontally and vertically 

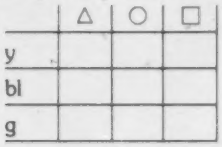
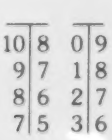
2–3	3–4
7–8 years	**8–9 years**
Given an object, student identifies another object which is different from original by one attribute, e.g., large, green, circle is different from large, red, circle in one way (color).	Given 2 (3, 4) objects, student identifies another object which is different from the original objects by one attribute, e.g., given a large red circle and a large yellow square, a large red square or a large yellow circle is different from the original blocks in one way.
Organize objects by 2 attributes on a matrix such as  	Use a matrix format to solve the problem above.
Identify sets organized by attribute(s) by referring to the attribute(s).	
Find missing object from a collection.	
Sort a collection of objects into a specified number of sets by abstracting an attribute, e.g., given a set of red or blue circles, triangles or squares, the student is instructed to organize the set into 3 groups so that the objects in each group have a common attribute.	
Select a set from a description of the one unifying attribute, including the complement (not), e.g., select the set of large blocks ("not" large blocks).	
Ordinal correspondence (3rd object from left) Order numbers to 100 Orders by 2 attributes (such as length and width)	One to one correspondence by size (pair 5 flowers of differing size with 5 vases of differing size, smallest with smallest, etc.) Order numbers to 1000
Extend patterns with 2 attributes such as color and shape Extend number patterns involving odd and even numbers (2, 5, 4, 7, 6, 9 . . .) Discover number patterns using tables such as:  	Recognize and extend patterns such as: ABACADAE, _ _ _ _ 4, 7, 10, 13, _ _ _ _ 3, 6, 9, 12, _ _ _ _ 2, 12, 22, 32, _ _ _ 8, 13, 18, 23, _ _ _ _ Recognize and extend geometric patterns involving rotation or flips.

Table 6 (continued)
Problem Solving

Grade	K–1	1–2
Age	**5–6 years**	**6–7 years**
GRAPHING	Line up two or three groups of real objects to determine more/less	Represent objects with blocks or counters to compare groups Read numbers of objects in pictographs Construct pictographs (one to one correspondence) Construct and read bar graphs (one to one correspondence) Use pictographs and bar graphs to compare groups as more, less, same.
ESTIMATION	Make guesses (which has more, less, how many blocks can be stacked before tower tumbles)	Make guesses (how many objects, more, less)
PROBABILITY and STATISTICS	Tally and discuss results of simple probability activities (tossing multicolored cubes, using spinners)	Perform simple probability activities, tally results and record on a graph (coins, unequal spinners)
NUMBER SENTENCES, WORD PROBLEMS APPLICATIONS	Discuss various numerical problems that arise naturally in class (milk money, attendance, bus loading)	Match number fact sentences to pictures 000 <=>3 + 2 = 5 △ △ Discuss problem situations that arise in class having numerical implications Use concrete objects to demonstrate and solve simple addition/subtraction story problems

2–3	3–4
7–8 years	**8–9 years**
Read and construct pictographs and bar graphs where information is labeled on two axes Collect data and construct graphs based upon student suggested topics	Read and construct line graphs from pictographs (horizontal and vertical lines only) Identify locations on a grid
Guess, fill and count to determine the number of objects (marbles, acorns, beads) it takes to fill 4 to 6 oz. transparent containers Experience adding and subtracting using multiples of 10. (27 + 10 = 37 so 27 + 9 = 36; 46 + 20 = 66 so 46 + 19 = 65; 52 − 20 = 32 so 52 − 19 = 33)	Guess, fill and count to determine the number of objects it takes to fill 8 to 10 oz. transparent containers Experience with rounding numbers Add and subtract using multiples of 10, 100 or dollars ($2.53 + $3.00 = $5.53 so $2.53 + $2.99 = $5.52; 432 − 200 = 232 so 432 − 198 = 234)
Make guesses of results, then experiment, tally, graph and compare results with prediction Discuss "likelihood" of various events Discuss "fairness" of games involving spinners, coin tossing Explore making subsets of one or two objects from sets of 2, 3 or 4 objects.	Make guesses of results, then experiment, tally, graph and compare results with prediction Use fractions to describe chance events Explore tree diagrams for combinations of events Explore arrangements of objects (number of monograms using F D and R, lining up 3 different books)
Write multiple number sentences that describe pictures (△ △ □ □ □) 4 + 6 = 10 (△ △ □ □ □) 2 + 2 + 3 + 3 = 10 10 − 6 = 4 Draw pictures to illustrate number sentences Use pictures to demonstrate and solve simple addition/subtraction story problems	Write number sentences that describe pictures (including multiplication and division sentences) Draw pictures to illustrate number sentences Create stories to describe number sentences from models Create stories with missing addend or sum in number sentences such as 3 + □ = 9, □ − 7 = 4 Use simple numerical data to write a question and a number sentence that will answer the question (10 cookies at noon, 2 cookies left at 2 p.m.) Translate addition/subtraction story problems into number sentences and solve Use objects and pictures to demonstrate and solve simple story problems involving multiplication Story problems involving ½, ⅓ and ¼

Table 6 (continued)
Problem Solving

Grade	4–5	5–6
Age	**9–10 years**	**10–11 years**
CLASSIFICATION	Given 2 (3, 4) objects, identify another object different from the original objects by 2 and 3 attributes. Select 2 sets and find the objects common to both (intersection of 2 sets). e.g., student selects all the red pieces and all the triangles and finds that the red triangles are in both sets.	Select 3 sets and find the objects common to all 3. (intersection of 3 sets) e.g., student selects all red pieces, triangles, and large pieces and finds that the large red triangle is common to all 3 sets. Select 2 sets and find all the objects that are in either one or the other set (union of 2 sets)
ORDERING (SERIATION)	Order numbers to 10,000	Order fractions with like denominators, like numerators, unlike denominators Order 1- and 2-place decimals
PATTERNING	Recognize and extend patterns (square numbers, triangular numbers, relations)	Recognize and extend patterns such as: 0, 1, 4, 9, 16, 25, __ __ __ __ 1, 3, 4, 7, 11, 18, __ __ __ __ State rules of patterns
GRAPHING	Construct and find information from pictographs and line graphs Construct tables (class attendance by day, temperature by hour) Identify locations on a rectangular grid	Interpret graphs including those with other than one-to-one correspondence (pictograph where each symbol represents 10 objects) Read tables and find information from them Construct time lines
ESTIMATION	Estimates of pictured groups (crowds of people, flocks of birds, bricks) Whole number operations—round and estimate then compute with paper and pencil or calculator	Estimation of solutions using rounding Use estimate, compute, check process for all calculations
PROBABILITY and STATISTICS	Make predictions of results, experiment, tally, graph and compare results with prediction Use concrete and pictorial materials to develop concept of average (mean) Exploration of probabilities expressed as fractions Construct frequency tables	Exploration of random samples Certainty and impossibility of events Make predictions based on sample data, verify with further experimentation Compute averages (means)

6–7 11–12 years	7–8 12–13 years
Classification skills are applied in problems involving the following strands: graphing, patterns, probability, and statistics.	Classification skills are applied in problems involving the following strands: graphing, patterns, probability, and statistics.
Order 2- and 3-place decimals Order integers	
Recognize and extend patterns such as: 3, 8, 5, 10, 7, 12, 9, 14, __ __ __ __ State rules of patterns	
Recognize patterns and trends from graphs, make predictions (weather, sales records, demographics) Experience with graphing points on a coordinate grid Construct and interpret circle graphs	Construct and interpret multiple line graphs from given or collected data Graph points on a coordinate grid
Use estimate, compute, check process for all calculations Use estimate, solve, evaluate reasonableness of process for all problems	Use estimate, compute, check process for all calculations Use estimate, solve, evaluate reasonableness of process for all problems
Definition of probability of an event Determine probabilities of: Events whose probabilities sum to 1 Independent events Determine mean, median and mode of a set of data Arrangements (permutations) of a set of objects Combinations of events (number of meals given a choice of 3 beverages, 2 main courses and 4 desserts)	Reinforcement of 6–7 topics

Table 6 (continued)
Problem Solving

GRADE	4–5	5–6
AGE	9–10 years	10–11 years
NUMBER SENTENCES, WORD PROBLEMS APPLICATIONS	Number sentences involving addition, subtraction, multiplication Use numerical data to generate questions, write number sentences and solve (5 buses, 8 people on each bus; purchases of $2.40 and $1.30, you have $5.00) Translate addition, subtraction, multiplication story problems into number sentences and solve Story problems involving money, time Use objects and pictures to demonstrate and solve simple story problems involving division Using a story create a two-step problem from models	Use numerical data to generate questions, write number sentences and solve Story problems involving addition, subtraction, multiplication and division of whole numbers Story problems involving money, time Story problems involving addition and subtraction of fractions and decimals Two-step problems Problems illustrating everyday applications of skills (food store purchases, hourly wage) Problems involving tables and graphs Problems involving measurement conversions Problems with extraneous information Problems with insufficient information Create flow charts for mathematical and nonmathematical activities
CALCULATORS AND COMPUTERS	Introduce calculator to check addition/subtraction problems	Introduce calculator for solving some story problems Exploratory interaction with computers Computer literary topics

6–7	7–8
11–12 years	**12–13 years**
Solution of open sentences ($3 \times n = 27$, $4.82 - \square = 2.75$)	Application problems involving:
Use formulas to solve problems	whole number, fraction and decimal computation
Application problems involving:	two or more steps
addition, subtraction, multiplication and division of whole numbers, fractions and decimals	measurement
two or more steps	proportions
measurement (buying carpet)	percents
proportions (unit pricing)	tables and graphs
percents (sales tax)	experiments and/or data collection
tables and graphs (train schedules, rainfall graphs)	Survival applications including:
experiments and/or data collection	gross salary and net pay
Logic problems	savings and checking accounts
Problems involving geometry	sales tax, discounts, markups, percent, gain or loss
Problems involving patterns	installment purchases
Problems with large numbers using a calculator	budgeting
Problems with extraneous or insufficient information	Nonroutine problems involving logic, patterns
Problems with numerous solutions	geometry
Create flowcharts for activities and processes	Problems with numerous solutions
	Create flowcharts for activities and processes
BASIC computer programming	BASIC computer programming
Continue computer literacy topics	Use of calculator to solve problems
Use of calculators to solve problems	

Table 7
Measurement/Geometry

GRADE	K–1	1–2
AGE	5–6 years	6–7 years
TIME	Sequence events using before, after, morning, afternoon, evening	Discuss and compare time for events (use hourglass, egg timer) Time (to hour) when events occur Time (to hour) on clockface Sequence calendar by seasons
MONEY	Identify pennies and nickels Trading games—pennies for candy, toys, crayons, etc., one-to-one Compare results before and after trading for more, less, same	Identify pennies, nickels, dimes Trading games with nickels and pennies
MEASUREMENT	Perception of attributes such as: big, small, tall, short, fat, thin, heavy, light, long, short Comparison of two objects on the basis of one attribute Linear measurement with nonstandard units (arm lengths, palm spreads, pencil lengths) Experience with balance scales (how many blocks balance a book) Experience with filling containers with sand, water, etc., using cups and other containers	Compare and order objects (3, 5, 8) on the basis of one attribute Estimate and measure length with non-standard units Use balance scale to compare weight of objects Estimate capacity, fill containers with objects or cupsful, count to check estimate (number of acorns to fill a coffee can) Estimate and measure area by covering with large squares or rectangles (Number of sheets of paper to cover door) Compare objects or events in terms of temperature (cooler, warmer, hotter, colder)
GEOMETRY	Experience with pattern blocks, tangrams, etc. making designs Recognize and identify circles, squares, triangles, rectangles and diamonds	Extend simple pattern block designs Construct designs using 3 tangram pieces Identify common geometric shape in everyday life Inside/outside Left/right

2–3	3–4
7–8 years	**8–9 years**
Read time from clock face (hour, ½ hour) Draw clockface representing time (hour, ½ hour) Days of week Months of year Use calendars	Read and represent time to nearest quarter hour Dates on a calendar
Trading with pennies, nickels, dimes Classroom store using pennies, nickels, dimes Value of sets of pennies, nickels, dimes	Value of sets of pennies, nickels, dimes and quarters (less than $1.00) Change for purchases less than 25¢ Money problems using dollars, dimes, and pennies to reinforce place value concept Dollar sign and decimal notation for money
Experiences estimating and measuring using various nonstandard tools (length of string, square, cube) Estimate and measure length in meters, centimeters, feet, inches Use balance scales and weights to estimate and measure weight in kilograms, pounds Estimate and measure capacity of a larger container in terms of a small container Read temperature on a thermometer	Estimate and measure length in meters, centimeters, decimeters, inches, feet, yards Estimate and measure weight in kilograms, grams, ounces, pounds Estimate and measure capacity in liters, ½ liters, ¼ liters, ounces, quarts Estimate and measure perimeter of polygons Measurement unit abbreviations
Construct designs using 5 tangram pieces, estimate and cover these designs using small triangles Determine relationships among patterns, blocks, tangram pieces Identify line segments, angles Identify sides and vertices of polygons	Compare common two- and three-dimensional objects (balls, boxes, cans, etc.) on the basis of size and shape Identify cubes, cones, spheres, cylinders

Table 7 (continued)
Measurement/Geometry

GRADE	4–5	5–6
AGE	9–10 years	10–11 years
TIME	Read and represent time to nearest minute Convert hours to minutes, minutes to hours Add and subtract time units Time equivalencies (7 days = 1 week, 24 hours = 1 day, etc.) Terms A.M., P.M., NOON, Midnight	Time to nearest second Time equivalents and conversions among years, weeks, days, hours, minutes and seconds. Elapsed time (past, future)
MONEY	Value of sets of coins and $1.00 bills Change for purchases less than $1.00	Value of sets of coins and bills to $25.00 Change for purchases to $10.00 Addition/subtraction of money amounts
MEASUREMENT	Estimate and measure length in meters, decimeters, centimeters, millimeters, yards, feet, inches, ½ inches Metric linear equivalents and conversion (1m = 1000mm = 100cm, 300cm = 3m, 40dm = 400cm = 4m) Customary linear equivalents and conversions (1 yard = 3 ft = 36 in., 48 in. = 4 ft) Estimate and measure weight in kilograms, grams, ounces and pounds Metric weight equivalents and conversions (1kg = 1000 gr) Customary weight equivalents and conversions (1 lb = 16 oz) Estimate and measure capacity in liters, ½ liters, milliliters, ounces, pints, quarts, gallons Metric capacity equivalents and conversions Customary capacity equivalents and conversions Estimate and measure rectangular area in square centimeters and square inches Freezing point and boiling point (Celsius and Fahrenheit)	Continued use of measurement tools to estimate and measure lengths, mass, capacity and temperature Reinforce metric equivalents and conversions among km, m, dm, cm, mm; kg, g, mg; kl, l, ml Reinforce metric equivalents and conversions among yards, feet, inches; pounds, ounces; gallons, quarts, pints Area and perimeter of squares and rectangles Use cubes to explore volume of rectangular solids Identify appropriate units for measuring length, area, volume, capacity, mass
GEOMETRY	Line symmetry Identify points, parallel lines, intersecting lines, diagonals Identify figures as congruent Identify right angles Identify quadrilaterals, pentagons, hexagons Construct circles, squares, triangles	Identify parallel and perpendicular lines Identify and discuss relationships among quadrilateral, parallelogram, rectangle, square Identify center, radius, diameter and circumference of circle Identify pentagons, hexagons, octagons Identify faces, edges, vertices of polyhedra Use protractor to measure angles

6–7	7–8
11–12 years	12–13 years

Metric prefixes milli, centi, deci, deka, hecto, kilo
Measure and convert among all metric units of length, capacity and mass
Relate cubic decimeter to liter to kilogram of water
Relate cubic centimeter to milliliter to gram of water
Measure and convert among customary units of inches, feet, yards, miles; ounces, cups, pints, quarts, gallons; ounces, pounds, tons.
Concepts of square and cubic measurement for area and volume

Reinforcement of 6–7 topics
Scale drawings

Identify right, acute and obtuse angles
Identify right, isosceles, and equilateral triangles
Use protractor to draw and/or measure angles, determine sum of the angles of a triangle
Construction of perpendicular bisectors, midpoints
Identify prisms, pyramids, cylinders, cones
Develop definition and concept of pi
Identify trapezoids and rhombi
Explore concepts of congruence, similarity and symmetry

Use formulas to find:
perimeters of squares and rectangles,
circumference and area of circles,
area of triangles, rectangles, squares, parallelograms, and trapezoids
volume of rectangular solid
Pythagorean theorem
Construction of congruent line segments, congruent angles, angle bisectors, perpendiculars to and from given lines

8 The 9–12 Program

A secondary school program of mathematics courses should provide four years of appropriate mathematics to meet the interests and abilities of all students. This is best accomplished through a varied set of course offerings, and levels within courses, designed to provide flexible sequences responsive to the diverse ability levels, academic plans and vocational aspirations of the student body.

A sufficient number of courses and course levels should be offered to meet these diverse needs and to provide mathematics commensurate with student ability. For example, a well-rounded high school program might entail the following courses:

remedial mathematics
general mathematics
consumer mathematics
basic algebra
informal geometry
business mathematics
shop/industrial mathematics
introductory computer science
algebra I
geometry
algebra II
advanced mathematics/precalculus
calculus
trigonometry
analytic geometry
probability and statistics
elementary logic
computer programming I
computer programming II

General considerations

In building an effective and responsive mathematics curriculum for students in grades 9–12, several important factors must be recognized.

There must be planned articulation between the grades K–8 program and the grades 9–12 program. At times, placement into 9th grade courses is done in a haphazard fashion. Often, information on the strengths and weaknesses of indi-

vidual students is not effectively transferred from the middle/junior high school to the high school, and 9th grade course objectives are drafted in isolation from the K–8 sequence upon which they should be built.

The solution to these common problems arises out of frequent and substantive communication among mathematics and guidance personnel and a carefully devised curriculum that addresses varying student needs and abilities. A district's process of monitoring student progress in mathematics, incorporating a record card and grade level post-testing, should be continued into the high school and used in conjunction with teacher recommendations to assist in the proper placement of students. For example, on the basis of 7th and 8th grade achievement, students should be directed into one of a range of suitable 9th grade courses, including: remedial mathematics, general mathematics, consumer mathematics, basic algebra, algebra or geometry. In all cases, course objectives for these courses should include appropriate overlapping with and extension of 8th grade objectives.

Also commonplace is the replacement of the sequence of specific student objectives that constitute a K–8 curriculum with a listing of "topics to be covered" that constitutes many 9–12 curricula. An effective 9–12 curriculum delineates course content in the form of specific student objectives that move sequentially within courses and between courses. For example, little direction is provided to either teachers or students when an algebra I curriculum lists: "Solve linear equations." Compare this with a set of student objectives such as:

- Given an equation, the student will solve it using the addition property of equality.
- Given an equation, the student will solve it using the multiplication property of equality.
- Given an equation involving several operations, a student will solve it using the properties of equalities in the appropriate order.
- Given an equation involving parentheses and like terms, the student will solve it using the appropriate order of operations (removing parentheses and combining like terms), and then using the properties of equality in the appropriate order.
- Given an equation for which the student has not learned a rote method (e.g., $\frac{1}{x} + \frac{3}{x} = 8$), the student solves the equation.

Another important consideration in designing course offerings and course objectives for a grade 9–12 program is finding ways to lessen dependence upon a single textbook and provide for greater use of additional and/or alternative instructional materials. For example, the curriculum of a consumer mathematics or business mathematics course should be supplemented with a range of nontextual publications and brochures often available within a local community. Similarly, practical and uncontrived applications of particular algebraic skills are rarely found in standard texts and must be gathered from other sources. Moreover, such courses as basic algebra or informal geometry do not have readily available texts and must be pieced together from a number of sources. In short, to the extent possible, a course curriculum must dictate the instructional materials rather than the more common situation where the textbook is the sole determinant of course objectives and activities.

Sample plans that address these considerations of carefully specifying course content and lessening dependence upon a single text are provided in Appendix F.

The widespread availability of hand-held calculators must also be considered when planning a secondary curriculum. While commonplace in trigonometry classes, the calculator remains an unwelcome intruder in many general or consumer mathematics classes. As curricular objectives are developed, serious thought must be given to the appropriate role of calculators in all mathematics classes. Certainly, as attention shifts from isolated computational abilities towards more general problem-solving abilities, the value of calculators and the opportunities for their effective use increase. For most students, this fundamental shift from knowing a particular computational skill to knowing when and how to apply the skill occurs prior to 9th grade, thereby making the calculator an appropriate and useful tool in high school classes.

The National Council of Teachers of Mathematics in its position statement on "Calculators in the Classroom"[6] suggests that the calculator be used:

- to encourage students to be inquisitive and creative as they experiment with mathematical ideas;
- to assist the individual to become a wiser consumer;
- to reinforce the learning of the basic number facts and properties in addition, subtraction, multiplication, and division;
- to develop the understanding of computational algorithms by repeated operations;
- to serve as a flexible "answer key" to verify the results of computation;
- to promote student independence in problem solving;
- to solve problems that previously have been too time-consuming or impractical to be done with paper and pencil;
- to formulate generalizations from patterns of numbers that are displayed, and
- to decrease the time needed to solve difficult computations.

Additional suggestions on calculators are addressed earlier under K–8 program considerations.

Finally, high school courses must be designed for maximum flexibility of sequencing. This entails keeping prerequisites to a minimum and discouraging lockstep tracking that prevents students from taking courses suitable to their changing needs and abilities. For example, a flexible and well-sequenced high school program should allow any student to take algebra I upon mastery of predefined basic skills, regardless of the student's grade level. Further, regardless of algebra experience, all students should have access to computer programming courses and appropriate consumer mathematics courses. Moreover, by providing alternatives to algebra II and/or advanced mathematics/precalculus, a school can strongly encourage four years of mathematics for all students, particularly for those juniors and seniors who have completed algebra I and geometry and would prefer alternatives to the traditional sequence. Such flexibility should encourage and allow for a four-year varied pattern of possible sequences like those illustrated in Table 8.

In summary, the developer of course curricula for grades 9–12 should assure that:

- course content is carefully delineated in the form of specific student objectives and not simply adopted verbatim from a particular text;
- appropriate problems and problem-solving activities are incorporated into all courses to illustrate and complement mathematics skills and concepts;
- computers and computer-related activities will be integrated wherever possible in all courses;
- skills and concepts are carefully sequenced throughout a series of courses to provide necessary overlapping for review and maintenance of skills;
- appropriate and distinct levels of specific courses are developed to meet varying student needs;
- the use of hand-held calculators is incorporated into all courses to reduce time-consuming computation, and
- the full array of course offerings allows every student to select an appropriate four-year sequence of mathematics.

Remedial mathematics

For a range of reasons, some students will inevitably arrive at the 9th grade seriously deficient in basic mathematics skills. These students, identified through local placement tests, the State's Ninth Grade Proficiency Test, or teachers' judgments, require an individualized remedial program that is based on a careful diagnosis of specific strengths and weaknesses. Whether provided through a

Table 8
9–12 Course Sequences (Sample)

Student	9th grade	10th grade	11th grade	12th grade
A	Geometry	Algebra II	Advanced Mathematics/ Precalculus	Calculus
B	Algebra I	Geometry	Algebra II	Advanced Mathematics/ Precalculus
C	Algebra I	Geometry	Algebra II	Trigonometry/ Probability and Statistics
D	Algebra I	Geometry	Computer Programming I	Elementary Logic
E	General Mathematics	Basic Algebra	Informal Geometry	Consumer Mathematics
F	Consumer Mathematics	Basic Algebra	Introductory Computer Science	Algebra I
G	Industrial Mathematics	Consumer Mathematics	Basic Algebra	Computer Programming I
H	Remedial Mathematics	Business Mathematics	Industrial Mathematics	Introductory Computer Science
I	Remedial Mathematics	Remedial Mathematics	Consumer Mathematics	Basic Algebra

resource room or math laboratory, a Title I-funded program, or as a part of the regular program, remedial mathematics courses must be offered to assure basic skills mastery by all students and to provide students with the necessary skills to progress into the mainstream high school mathematics program.

To be effective, remedial mathematics classes should:

- be limited in class size to five to ten students;
- be individualized to the degree possible on the basis of each student's specific weaknesses;
- be structured around an announced list of specific objectives that students must master;
- relate the mathematics skills to practical applications and everyday situations whenever possible, and
- strive to incorporate alternative approaches and activities including the continued use of concrete and representational materials.

One approach to determining a set of appropriate objectives for remedial mathematics programs is through the adoption of the 37 objectives presently used for developing Connecticut's Ninth Grade Proficiency Test (see Appendix G). These objectives provide a core curriculum of computation skills, concept understanding and word problem applications. Moreover, the objectives can be used to create specific skill packets and other materials that facilitate individual and small group instruction.

Basic and vocational courses

Basic mathematics and vocationally oriented mathematics courses are offered to provide students—in their future roles as workers and consumers—with a full understanding of the practical and essential uses of basic mathematics. As such, these courses must be developed to include:

- a review and reinforcement of previously learned skills and concepts,
- a focus on the applications of these skills and concepts in a variety of real situations, and
- where appropriate, exposure to more advanced mathematics as a means to encourage further study of mathematics.

In developing these basic and vocational mathematics courses, the primary curricular goal must be placing mathematics skills in practical contexts. For example, while an appropriate remedial or general mathematics course objective might be "given two decimal numbers, the student will find the sum and difference," this same skill is transferred into a consumer mathematics class in the form of "given a beginning balance and a series of deposits and withdrawals, the student will keep a running balance on a checking account." In this fashion, remedial and general mathematics courses are conceptualized as opportunities for slower learners to master skills typically found in grades 6 through 8 while consumer-oriented and vocationally oriented courses shall entail the development and application of these skills. In practice, however, this distinction may blur as skills and applications are blended together in all courses.

The following topics are replete with mathematical applications that can form the core of basic and vocational mathematics courses.

Business and consumer topics

- automobile costs
- catalog shopping
- checking accounts
- counting calories
- credit card, credit unions, loan costs
- discounts, markups
- income taxes
- installment purchases
- insurance costs
- interest, mortgage, tax tables
- payroll calculations, commissions
- proportions to find unit prices
- reading charts and graphs
- recipes
- sales tax
- savings accounts, interest calculations
- telephone charges
- utility bills, reading meters
- water, gas electricity, fuel oil costs

Industrial and career topics

- air and water pollution
- area, volume calculations
- basic trigonometry
- billing jobs
- calculating dimensions
- direct and inverse proportion
- furniture or building construction
- geometric constructions
- inside and outside diameter calculations
- inside and outside diameter calculations
- measurement of all types
- medical dosage calculations
- Ohm's law
- purchasing wood or metal stock
- rates of revolution, cutting speeds
- scale drawings
- screws, bolts, threads
- tolerances
- using formulas

In addition, motivating examples that employ a broad range of mathematics skills can be drawn from typical work experiences of the plumber, electrician, carpenter, farmer, dietician, advertising executive, retail store or restaurant employee or manager, mechanic, insurance agent or adjuster, architect, bookkeeper or accountant, truck driver, forestry land manager, or bank teller. Obviously this list can be expanded to show the essential role that mathematics plays in nearly all possible career choices and provides an opportunity for infusing career education into mathematical courses.

Sports and gambling are two other areas replete with mathematics in the form of statistics, percentages and probabilities.

Courses built upon units chosen from the above topics provide a mathematics curriculum in which computational skills and conceptual understandings are reinforced within the context of their applications and employed as a means to solve problems—certainly a major goal of all mathematics programs.

An additional component of the high school mathematics program for the noncollege-bound student includes an introduction to algebra and geometry. Such courses as basic algebra or informal geometry, although often overlooked, provide an essential bridge between the basic/vocational courses described above and the traditional college preparatory program. Probably best suited as one semester offerings, these courses address the significant need for certain algebraic and geometric skills and understandings in many careers that do not require post-secondary study. Further, the inclusion of such courses increases the probability that students can be enticed into continuing beyond the commonly required two years of mathematics.

A basic algebra course, distinctly different from the more common pre-algebra, should be conceived as a self-contained entity entailing the mastery of a specific set of objectives rather than serving as exposure and prelude to the traditional first year algebra course. As such, the course should be well grounded in concrete embodiments of concepts and applications and might be built around a core that includes:

- integers
- order of operations and evaluating algebraic expressions
- linear equations with an emphasis on formulas and using linear equations to solve realistic (career-oriented) problems
- proportions (direct and inverse) including percent problems and other applications
- graphing linear relations
- exponents and square roots
- areas, volumes, the pythagorean theorem and related geometric applications
- problems involving belts, pulleys, gears, measurement

In this fashion, the course would omit such familiar algebraic topics as sets, axioms, contrived word problems, inequalities, and most factoring and algebraic fraction work.

Similarly, an informal geometry course, making use of geoboards, tangrams, pentominoes, compass and straightedge, and other devices, should be developed to include a specific set of geometric skills and understandings. While stopping far short of formal deductive proof and omitting many of the traditional geometry course theorems, such a course might be built around a core that includes:

- two- and three-dimensional geometric figures and shapes
- parallelism and perpendicularity
- the concepts of congruence and similarity including scale drawings
- constructions via paper folding as well as compass and straightedge
- basic coordinate geometry
- area and volume
- geometric patterns
- the right triangle
- the circle

Finally, an introductory computer science course should be offered as part of this sequence. An excellent list of objectives directed at developing essential computer literacy appears in the February 1980, issue of *Mathematics Teacher*.[7] These objectives clearly outline course content that all students should be exposed to.

Traditional college-bound courses

The algebra I to calculus or precollege sequence is probably the least controversial and most common part of the grades 9–12 curriculum. These courses are the most accessible in terms of readily available textbooks and the most standardized in terms of an accepted course of content.

Within this uniformity, however, some prefer a sequence of algebra I followed by geometry and then algebra II. Others offer algebra I, algebra II and then geometry. Still others are beginning to experiment with a three-year integrated approach to the same content. Likewise, some prefer Euclidean plane geometry while others turn to a transformational approach. Similarly, what is called advanced algebra for some turns out to be precalculus for others, and while some incorporate trigonometry into algebra II, others leave it for a following year. In most cases, despite differences of course title and variations in sequence, the core curriculum of specific skills, concepts and understandings changes little.

Thus, developing curriculum for the traditional, precollege sequence of algebra I, geometry, algebra II, advanced algebra and calculus begins with a detailed sequence of skills and concepts that a curriculum committee allocates to specific courses and in turn expands into appropriate course objectives.

In developing these course curricula, several key issues should be addressed.

- Course content must allow for review and reinforcement of basic skills. Periodic work with fractions, percents, and other areas of common difficulty should be integrated into all courses.
- Course objectives should recognize and account for various embodiments of concepts and approaches to skills. All too often, texts provide only one approach and an effective curriculum will supplement this with others.
- To the degree possible, a set of practical problems and applications should be interwoven into all courses for the purpose of tying isolated skills together and continually emphasizing the underlying utility of studying mathematics.
- Similarly, to the degree possible, each course should provide an opportunity for students to discover and understand the basic inherent structure of mathematics, apart from its practical value.
- Often, to meet varying student needs and abilities, two levels (in addition to an honors level) of algebra I, II and geometry should be considered, with careful thought given to the differences in content, texts, tests, and supplementary materials between levels.

Electives

An array of elective course offerings helps to assure that the needs and interests of all students are met and encourages more students to continue beyond minimum mathematics requirements. Whether chosen in addition to another mathematics course or in place of it, elective courses allow for the specialization of content to suit the interests of both students and teachers and provide alternatives to traditional course sequences.

In planning and developing elective courses, prerequisites should be determined with care. While elective courses should build upon the skills of prior courses, there is no reason to establish an algebra II prerequisite, for example, for many computer programming courses. Similarly, a consumer mathematics elective, specifically tailored for juniors and seniors, might be offered without any prerequisite.

Among the electives, usually offered as one semester courses, that broaden the range of mathematics department offerings and help to serve a wider spectrum of the student body are:

- analytic geometry
- computer programming I
- computer programming II
- elementary logic
- probability and statistics
- trigonometry

9 Interdisciplinary Considerations

Mathematics is a basic skill that has value as a tool in many other disciplines. By bringing examples and applications from other disciplines into the mathematics curriculum, the common isolation of mathematics is reduced and the curriculum is enriched with essential interdisciplinary experiences. Moreover, subject areas, such as career and consumer education can be readily infused into mathematics curriculum.

The following, arranged by discipline, is a partial listing of topics that can be used to develop individual lessons or units.

Art

- computer art
- dynamic symmetries
- geometric designs
- geometric figures in witchcraft
- golden mean
- graphics of M.C. Escher
- kinetic art
- movement and visual sequence in animation and moviemaking
- optical illusions
- ornamental, symmetrical designs
- perspective

Business

- bookkeeping, accounting
- cost estimation
- decision making and matrix algebra
- insurance
- inventory programs
- markups, discounts
- patterns of trade
- payroll
- taxes

Driver Education

- centrifugal force
- coefficient of friction
- force impact
- gravity
- kinetic impact

Language Arts (including reading)

- analysis/study of language
- language in logic
- logic in children's literature (Lewis Carroll, A.A. Milne)
- mathematical fantasy (*Flatland*)
- measurement of reading speed and comprehension
- systems of symbolization as a means for communication
- transformational grammar

Health Education

- calculation of blood alcohol level related to drinking
- computations in vital statistics
- graphs and charts
- Jellinek formula in estimation of alcoholism rates
- nutrition (diets, calories, amounts and percents of nutrients)
- profit ratios from opium poppy to street heroin
- psychological tests
- sets
- time as a measure of intoxication

Industrial Arts

- mathematics of linkages
- measurement
- ratio, proportion, percent, fractions, decimals
- scale drawings
- study of devices used to gain mechanical advantage
- work-efficiency study

Music

- composite sounds
- musical forms
- physics of sound
- scales and tonality

Physical Education

- analysis of movements, types of levers, etc.
- areas of court games (volleyball, tennis, etc.)
- games "Counting off," "Ten Little Indians," "One, two, three, O'Leary"
- team grouping

Science (including astronomy, biology, physics, chemistry)

- balancing equations, simple proportions
- coordinate systems for the skies
- crystalline structure of biolgoically important molecules
- decibel scale
- energy use
- mathematics of heredity, statistics, probability
- mathematics of the telescope
- measurement
- motions of the members of the solar system
- pattern recognition by insects
- estimation
- examples of Fibonacci sequence and equiangular spiral in nature
- first degree equations for Newton's second law, Ohm's Law
- insect and bird navigation
- inverse square relationship for point sources of light, heat, gravitational attraction, electical attraction and repulsion
- population studies, plant and animal
- second degree equations for kinetic energy
- sunspot cycles
- surfaces of minimum area
- vernier
- weather phenomena—rainbows, tornadoes

Social Studies (including anthropology, geography, history)

- biographical studies of eminent mathematicians
- dating of artifacts by radioactive isotopes
- feeding people and world hunger
- history of machinery
- history of measuring devices and systems of numeration
- map making and map projection
- mathematics in the newspaper
- measurement and statistical study of biological characteristics of human beings
- opinion poll sampling
- population studies
- development of mathematical concepts and skills in a given culture
- logical thinking patterns of different cultures
- tables, graphs and charts

Additional interdisciplinary applications can be found in *Application of Secondary School Mathematics* and *Application in School Mathematics*, both from the National Council of Teachers of Mathematics; cocurricular applications of mathematics and physical education appear in *The Olympics: An Educational Opportunity, Enrichment Units, K–6* and *7–9*, (see Appendix C).

Appendix A
Statewide Goals for Education

From Connecticut's *Comprehensive Plan for Elementary and Secondary Education, 1980–1985*

GOAL ONE

Motivation to Learn

To realize their potential to learn, students must be highly motivated.

Therefore:

Connecticut public school students will develop strong motivation by responding to the high expectations of their parents, teachers and school administrators; by understanding and striving to fulfill personal aspirations; and by developing the positive feelings of self worth which contribute to responsible behavior and personal growth, health and safety.

GOAL TWO

Mastery of the Basic Skills

Proficiency in the basic skills is essential for acquiring knowledge and for success in our society.

Therefore:

Connecticut public school students will, to their full potential, learn to communicate effectively in speech and writing; read with understanding; acquire knowledge of and ability in mathematics; and strengthen decision-making skills.

GOAL THREE

Acquisition of Knowledge

Acquiring knowledge leads to fuller realization of individual potential and contributes to responsible citizenship.

Therefore:

Connecticut public school students will acquire the knowledge of science, mathematics, social studies, the arts, literature and languages which leads to an understanding and appreciation of the values and the intellectual and artistic achievements of their culture and other cultures; and will take full advantage of opportunities to explore, develop and express their own uniqueness and creativity.

GOAL FOUR

Competence in Life Skills

Students are challenged to function successfully in multiple roles: as citizens, family members, parents, producers and consumers.

Therefore:

Connecticut public school students who complete secondary level studies will have the ability to make informed career choices; understand the responsibilities of family membership and parenthood; be prepared to undertake the responsibilities of citizenship in their communities, in the state, in the nation and in the world; and have the skills, knowledge and competence required for success in meaningful employment, or be qualified to enter postsecondary education.

GOAL FIVE

Understanding Society's Values

To be responsible citizens and contribute to positive change, students must understand and respect the underlying values of this society.

Therefore:

Connecticut public school students will appreciate diversity and understand the inherent strengths in a pluralistic society; they will understand and respond to the vital need for order under law; they will acquire the knowledge necessary to live in harmony with the environment, and actively practice conservation of natural resources, and they will respect the humanity they share with other people.

Appendix B
Legislation

The series of guides to curriculum development published in 1981 by the State of Connecticut Board of Education is consistent with the provisions of Sections 10-4 and 10-16b (or P.A. 79-128) of the Connecticut General Statutes.

Section 10-4. Duties of Board. (a) . . . shall prepare such courses of study and publish such curriculum guides . . . as it determines are necessary to assist school districts to carry out the duties prescribed by law . . . .

Section 10-16b. Prescribed courses of study. (a) In the public schools the program of instruction offered shall include at least the following subject matter, as taught by legally qualified teachers: the arts; career education; consumer education; health and safety; language arts, including reading, writing, grammar, speaking and spelling; mathematics; physical education; science; social studies, including, but not limited to, citizenship, economics, geography, government and history; and in addition, on at least the secondary level, one or more foreign languages and vocational education.

(b) Each local and regional board of education shall on September 1, 1982, and annually thereafter at such time and in such manner as the commissioner of education shall request, attest to the state board of education that such local or regional board of education offers at least the program of instruction required pursuant to this section, and that such program of instruction is planned, ongoing and systematic.

(c) The state board of education shall make available curriculum materials and such other materials as may assist local and regional boards of education in developing instructional programs pursuant to this section.

Appendix C
Resources

Among the many sources of information relevant to curriculum development in mathematics are publications, ERIC searches (Educational Resources Information Center with 16 regional clearinghouses), professional associations (often the disseminators of professional journals), Connecticut's regional education service centers and centers devoted to mathematics at institutions of higher education in the state. The following list contains selected references; it is not a comprehensive list of available resources.

Publications

Elementary Methods and Materials

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ment Units, K–6. Colorado Springs, CO 80909.
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Professional Associations

Associated Teachers of Mathematics in Connecticut (ATOMIC)
Box 234
Avon, CT 06001

Association of Teachers of Mathematics in New England (ATMNE)
University of New Hampshire (changes annually)
Durham, New Hampshire 03104

Publication: *New England Mathematics Journal*

National Council of Teachers of Mathematics (NCTM)
1906 Association Drive
Reston, VA 22091

Publications: *Arithmetic Teacher*
Journal for Research in Mathematics
Mathematics Teacher

Regional Educational Service Centers

Area Cooperative Education Services (ACES)
800 Dixwell Avenue
New Haven, CT 06511

Capitol Regional Education Council (CREC)
212 King Philip Drive
West Hartford, CT 06117

Cooperative Educational Services (CES)
11 Allen Road
Norwalk, CT 06852

Eastern Connecticut Regional Educational Service Center (EASTCONN)
R.R. 2
Willimantic, CT 06226

Long-Range Educational Assistance for Regional Needs (LEARN)
P.O. Box 220
East Lyme, CT 06333

Regional Educational Services Concept through United Effort (RESCUE)
R.R. 2, Goshen Road
Litchfield, CT 06759

Higher Education

Mathematics Education Center
University of Connecticut
P.O. Box U-7
Storrs, CT 06268

Curriculum Laboratory
Elihu Burritt Library
Central Connecticut State College
New Britain, CT 06050

Other Sources of Information

Calculator Information Center
1200 Chambers Road
Room 201
Columbus, OH 43212

Connecticut Facilitators Project
ACES
800 Dixwell Avenue
New Haven, CT 06511
(dissemination of validated
Title IV mathematics
programs)

Minnesota Educational Computing
Consortium (MECC)
Saint Paul, MN 55113

Technical Education Resource Center
(TERC)
8 Eliot Street
Cambridge, MA 02138

Appendix D
Sample Curriculum Guide Formats

SAMPLE 1

Grade: 2–3

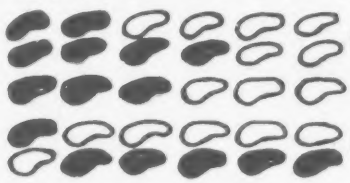
Unit/Strand: Number Work/Addition, Problem Solving/Probability, Problem Solving/Graphing

Objective(s): Learn basic combinations of sum. Experience predicting combinations. Construct graphs.

Learning Experiences/Activities:

Materials: Red/White Beans (Large lima beans spray painted red on one side) Record sheets.

Activity: Toss (6) beans (Experiences with other sums to 10)



(Combinations 2,4)

(Combination 3,3)

(Combinations 1,5)

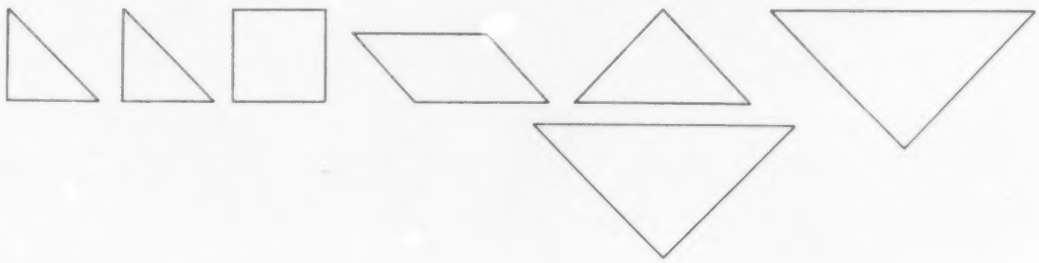
Recording Procedure: Graph results.

Sum 6			
0,6	1,5	2,4	3,3

SAMPLE 2

Grade: 2–3, 3–4
Unit/Strand: Measurement/Geometry
Objective(s): Experiences for seeing relationships of geometric shapes.
Learning Experiences/Activities:

Materials: Colored construction paper cut into Tangram pieces.



Use sets of 3 pieces. Make designs/shapes. Paste. Make booklets. Make puzzle cards or designs/shapes (outline the shapes). Make the shapes with pieces.

Use 4 pieces.

Use 5 pieces.

SAMPLE 3

Grade: 3–4
Unit/Strand: Number Work/Multiplication
Objective(s): Experience for finding factors of prime and composite numbers. Learn basic multiplication combinations.

Learning Experiences/Activities:

Materials: One inch squares of colored construction paper

Activity: Use squares to form rectangles of 3, 4, 5 . . . 24 squares. Establish that all squares are rectangles but all rectangles are not squares.

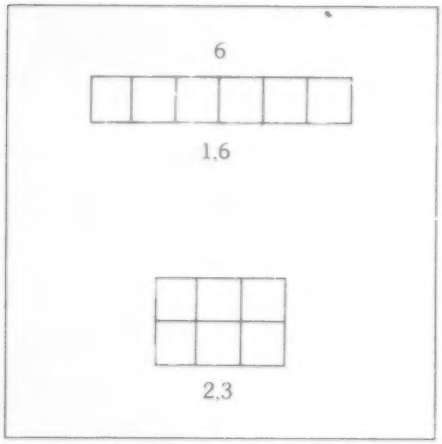
Rule: and are the same shape

Paste rectangles for each total (product) on separate sheets.

Organize pages for those with one rectangle to make booklet of prime numbers.

Organize pages with more than one rectangle to make booklet of composite numbers.

Label factors.



Appendix E
Excerpts of Sample Student Record Card

SAMPLE 1 Check (✓) appropriate grade(s) at which mastery is attained.

Grades K	1	2	3	4	5	
						SETS
						1. recognition: set, member
						2. one to one matching
						3. joining sets
						4. separating sets
						5. comparison: more, fewer
						6. number of sets to 18
						7. equivalent sets
						8. non-equivalent sets
						9. empty set
						10. brackets
						11. commas
						12. capital letters to name sets
						13. n ( ) notation
						14. equal
						NUMBER AND NUMERATION
						1. count objects to 10
						2. matching numeral to number of set
						3. reads 0–10
						4. writes 0–10
						5. counts to 50
						6. counts to 100
						7. reads to 100

Grades K	1	2	3	4	5	Number and Numeration, cont.
						8. writes to 100
						9. ordinals to fifth
						10. counts to 999
						11. reads to 999
						12. writes to 999
						13. counting by 2's, 5's, and 10's
						14. number words zero to ten
						15. number words zero to twenty
						16. ordinals to eighth
						17. place value to tens
						18. place value to hundreds
						19. expanded notation—ones, tens
						20. expanded notation—ones, tens, hundreds
						21. renaming through tens
						22. count to 9,999
						23. read to 9,999
						24. write to 9,999
						25. ordinals to twentieth
						26. number words to hundred
						27. place value to thousands
						28. expanded notation to hundreds

SAMPLE 2

EXCERPT OF SAMPLE STUDENT RECORD CARD TIED DIRECTLY TO CRITERION-REFERENCED TESTS

Source: Danbury (CT) Public Schools

Test 10	**Multiplication Basic Facts** Multiply two one-digit numbers (memorizing the multiplication tables through the nines)	**Level III** Mastery ☐ Date _____
Test 11	**Subtraction** Subtract two & three digit numbers where the tens are regrouped to get more ones (borrowing from tens only) Subtract two & four digit numbers where the 100's only are regrouped Subtract two & three digit numbers where both the 10's & 100's are regrouped	**Level III** Mastery ☐ Date _____
Test 12	**Multiplication** Multiply a number by a one-digit number, no carrying Multiply a number by a one-digit number, with carrying into ten's Multiply a two-digit number by a one-digit number with carrying into the ten's & hundreds places	**Level III** Mastery ☐ Date _____
Test 13	**Division** Divide a one-digit number into a one-digit number (no remainder) Divide a one-digit or two-digit number by a one-digit number with one number in the quotient, with remainder	**Level III** Mastery ☐ Date _____

Test 14 **Multiplication** **Level IV**

Multiply any number by a one-digit number
Multiply a two-digit number by a two-digit number
Multiply any number by a two-digit number
Multiply with multiples of ten (introduction of zero as a place holder)

Mastery ☐
Date _____

Test 15 **Division** **Level IV**

Divide a two-digit number by a one-digit number with a one-digit number or 10 in the quotient (basic fact through $9\overline{)90}$, no remainder
Divide a two or three digit number by a one-digit number with a quotient of one, two, or three digits

Mastery ☐
Date _____

Test 16 **Division Advanced** **Level IV**

Divide a two or three digit number by a one-digit number with a remainder

Mastery ☐
Date _____

Test 17 **Word Problems** **Level IV**

Add, subtract, multiply & divide to solve word problems
Solve problem involving money
Solve word problems using information from a chart, drawing, graph, or map
Use two arithmetic steps to solve word problems

Mastery ☐
Date _____

Test 18 **Basic Fractions** **Level IV**

Identify the object in a set as indicated by a given fraction
Write fractions in simple form
Change fractions to mixed numbers

Mastery ☐
Date _____

Appendix F
Sample Instructional Plans

Source: South Windsor (CT) High School

SAMPLE 1

LESSON PLAN OUTLINE **COURSE:** Consumer Mathematics **UNIT:** Banking

Topic: Preparing a checking account "Statement of Account"
Materials: Packets of checks, bank statement worksheets
Resources: Teacher and Student—Textbook Lesson
Expected Learning Outcome:
- Be able to read a checking account statement.
- Recognize the pitfalls and benefits of using a checking account.

Teaching Objectives:

* Symbols/Terminology
Recognize—deposit, credit, charges, fees, balance, reconcile, outstanding checks, statement of account

* Concepts
Balance = Deposits + Credits—Charges at any given time.

* Processes
Preparing a statement of account from the processed checks.
Reconciling the statement (details in the next lesson).

Procedure:
- Issue each student a packet of checks written by a student in another section of the course.
- Have each student remove one or two checks from the packet to create an outstanding balance.
- Students then determine the various fees: service charge, unsigned check fee, miscellaneous fees, overdrawn account fee, etc.
- Preparation of the bank statement of account. Use the worksheets here.

Criterion Performance:

Assignment:
Use your packet of checks.
Prepare the statement of account for student XYZ.

SAMPLE 2

LESSON PLAN OUTLINE **COURSE:** Algebra 2 **UNIT:** Quadratic Functions

Topic: Computerlab demo: Solution of quadratic equations using the quadratic formula

Materials: Microcomputer with monitor, program cassette, program flowchart sheets, program listing sheets

Resources: Teacher—In-service computer materials; Student—Flowchart sheets, listing sheets

Expected Learning Outcome: Awareness of

- constructing a flowchart for the program
- writing a program from the flowchart
- providing for computer printout of complex numbers

Teaching Objectives:

* Symbols/Terminology

Recognize—flowchart, discriminant facsimile (as general vocabulary word)

* Concepts

- A computer program for solving a quadratic equation having integral coefficients. Must permit any integers for a, b, and c.
- The computer prints real numbers in decimal form.
- For solutions that are complex numbers, the program should provide for the construction of facsimiles of the numbers in either the a + bi or (a,b) form.

* Processes

- Constructing a flowchart for the program
- Writing a program from the flowchart.
- Testing the program with test data.

Procedure:

- Q and A interaction with the students as the computer is made ready to run the program: loading the program, etc.
- Quick demo of the program by the teacher. Use a = 1, b = 5, c = 6.
- Issue flowchart sheets and develop the logic of the program.
- Issue program listing sheets and correlate each statement with the flowchart.
- Have each student enter one set of test data from the previous day's assignment as a check for correctness.

Criterion Performance:

Assignment:

Given a flowchart, write a program to solve a quadratic equation.

Special Directives:

Set aside your program listing sheet until you have written a full program. Compare. Redraft your program if necessary, keeping the listing set aside.

SAMPLE 3

LESSON PLAN OUTLINE **COURSE:** Geometry **UNIT:** Areas of polygons and circles

Unit: Areas of polygons and circles
Topic: Special properties of a regular polygon
Materials: Compasses, straightedge, infosheet 15–5
Resources: Teacher and Student—Textbook Ch. 15 Sec. 5
Expected Learning Outcome: Find the measure of the apothem, radius, and central angle of a regular triangle, quadrilateral, hexagon.

g Objectives:
ɔymbols/Terminology
Define—Center of a regular polygon apothem
Radius of a regular polygon
Central angle of a regular polygon
Recognize—n—number of vertices s—measure of a side
* Concepts
Regular polygons may be inserted in or circumscribed by a circle.
Properties of right triangles may be used to determine the apothem and radius of these polygons.
We can develop particular formulas to give the apothem and radius of regular triangles, quadrilaterals, and hexagons.
The measure of the central angle of a regular polygon may be found by simple division.
* Processes
Use $m\angle AOC = \frac{360}{n}$ tc find measure of the central angle.
Use a = f(s) to find the measure of the apothem.
Use r = f(s) to find the measure of the radius.

Procedure:
- Show projectuals of several different regular polygons.
- Define center, radius, apothem and central angle of a regular polygon.
- Interactively develop with students a formula for finding the central angle of a regular polygon
- Interactively develop with students formulas for the apothem and radius as functions of the side of the polygon: a = f(s), r = f(s). Use infosheet 15–5 here.

Criterion Performance:
Assignment:
Textbook p. 253 Exs. 1–10
Special Directives:
Be sure to draw and label an appropriate diagram for exercises 1–9.

Appendix G
Education Evaluation and Remedial Assistance (EERA) Ninth Grade Proficiency Test Objectives

The following objectives or skills are intended to represent a broad sample of mathematics skills and understandings. They attempt to address the broad categories of computation skills, concepts, and problem-solving. However, given the constraints of a one-hour examination, they are not meant to be, nor could they be, an exhaustive list of all mathematics topics taught prior to testing.

1. Add whole numbers.
2. Subtract whole numbers.
3. Multiply whole numbers.
4. Divide whole numbers (without remainders).
5. Add fractions and/or mixed numbers.
6. Subtract fractions and/or mixed numbers.
7. Multiply fractions and/or mixed numbers.
8. Divide fractions and/or mixed numbers.
9. Add decimal numbers.
10. Subtract decimal numbers.
11. Multiply decimal numbers.
12. Divide decimal numbers.
13. Find a percent of a given whole number.
14. Find what percent one whole number is of another whole number.
15. Convert fractions, decimals, and percents to equivalents.
16. Order unit fractions or decimal numbers.
17. Identify the numeric form of a given whole number written in words.
18. Identify the place value of a digit in a given number.
19. Name a ratio given two quantities.
20. Recognize a given pair of lines as parallel, perpendicular, or intersecting.
21. Identify the fractional equivalent of the shaded portion of a given pictorial representation.
22. Select the most appropriate unit of measure for a given task.
23. Find the perimeter of a common geometric figure (triangle, rectangle, square).

24. Find the area of a common geometric figure (circle, triangle, rectangle, square).
25. Solve for a variable in a given formula.
26. Solve a problem involving whole numbers.
27. Solve a problem involving fractions.
28. Solve a problem involving decimals.
29. Solve a problem involving percents.
30. Read and interpret a table, chart, or graph.
31. Read and interpret a map drawn to scale.
32. Find equivalent linear measures (English or metric).
33. Find equivalent measures of weight (mass) and capacity (English or metric).
34. Solve a problem involving time.
35. Find the average of a set of whole numbers.
36. Approximate a reasonable answer to a given problem.
37. Identify the correct number sentence to solve a problem.

Appendix H
ERIC Clearinghouse Searches in Mathematics

Available at the School Services Unit, ACES,
800 Dixwell Avenue, New Haven, CT 06511 for use at the site.

Code Number	Title
770 143-2	Criterion Referenced Tests—K–5, Mathematics
770 543-1+	Mathematics–High School, Individualized Instruction and Tutoring Programs
770 576-2	Evaluation of Mathematics, K–12
770 708-1+	Elementary School Mathematics
770 857-2	Cost Analysis on Elementary Mathematics Programs
770 870-2	Retention of Skills in Reading and Mathematics: Elementary Grades
770 943-1+	Mathematics Performance Objectives, Grades 9 and 10
770 951-2	Mathematics Programs for Low Ability High School Students
770 978-1	Content Reading in Mathematics and Science—Middle School
770 990-1+	Mathematics Instruction Scope and Sequence, Grades K–8
780 526-1+	Mastery Learning in Mathematics
780 107-1+	Adult Basic Education Mathematics Programs
780 121-2	Effective Mathematics Programs, K–12
780 131-1+	Individualized Instruction in Mathematics Programs: Grades 7–12
780 142-1+	Teaching Styles Mathematics
780 218-1+	Ratios and Proportions
780 220-1+	Self Concept and Academic Achievement in Mathematics at the Middle School
780 517-1+	Competencies in Mathematics
780 576-1+	Curriculum Guides: K–6 (Mathematics, Reading, Language Arts, Science and Social Studies)
780 707-2	Program Evaluation of Elementary Mathematics
780 747-1+	Screening Tests, Remedial Programs—Elementary Mathematics
780 826-1+	Middle School Mathematics Curriculum
780 890-2	Science and Mathematics for the Gifted and Talented: Junior High Level

780 947-2	Computer Assisted Instruction for Mathematics and Science, (Grades 7–12)
790 003-1+	Reading and Mathematics Competency Tests, Elementary Level
790 004-1+	Curriculum for Language Handicapped Children, Pre K–6
790 262-1+	Teaching Problem Solving in Secondary School Mathematics
790 303-1+	Dyscalculia
790 307-1	Evaluation of Record Management Systems in Individualized Mathematics Programs, K–6
790 357-1	Predictors of Female Success in Mathematics and Science
790 437-1+	Homogeneous vs. Heterogeneous Grouping in Middle School Mathematics
790 449-1	Basic Skills Instruction Using Visual Media
790 561-1+	Curriculum Materials in Language Arts and Mathematics for Mainstreamed LD and EMR Secondary School Students
790 627-1	Career Education in Academic Subjects, 9–12
790 668-1+	Piaget and Mathematics Instruction
790 836-1+	Predicting Success in Algebra and Geometry
790 940-1+	Remedial Programs in Junior High Schools
790 955-1	Teaching Methods and Mathematics Achievement
790 992-1+	Integration of Mathematics Curriculum and Job Skills
790 1018-1+	Mathematics Placement Tests as Predictors of Success in Mathematics Courses
800 301-1	Mathematics and Science Anxiety: Sex Equity and Mathematics and Science
800 344-2	Sex Differences and Mathematics Achievement
800 404-1	Infusion of Mathematics in Career Education, K–9
800 415-1+	Evaluative Testing for Honors Mathematics at the Junior High Level
800 443-1+	Business Mathematics Curriculum Objectives
800 511-1	Effects of Calculators in the Classroom, K–12
800 584-1+	Mathematics for Machine Tools

Appendix I
Standards for Successful Mathematics Programs

Source: "Standards for Successful Mathematics Program," Maryland State Department of Education, 1978

The following Standards have been developed to assist principals and staff in the evaluation of school mathematics programs. *All* 21 Standards are attainable. Therefore, attainment of *all* 21 will indicate a successful mathematics program.

No attempt has been made to define "attained." The decision of whether the Standard is "attained" or "not attained" must be based on careful and thoughtful discussion, including documentation wherever possible, by all members of the school staff associated with mathematics instruction.

INSTRUCTION

STUDENT ACHIEVEMENT DEPENDS ON LONG-RANGE, SHORT-RANGE, AND DAILY PLANNING OF CLASSROOM ACTIVITIES BY THE TEACHER. THESE ACTIVITIES SHOULD BE AN OUTGROWTH OF INSTRUCTIONAL TECHNIQUES AND POLICIES AIMED TOWARD STUDENT ATTAINMENT OF CLEARLY DEFINED OBJECTIVES.

STANDARD 1 *Student attainment of clearly defined objectives is a primary goal of the instructional program.*

STANDARD 2 *Assessment devices are administered by the teacher and data obtained are utilized in planning instructional activities for individual students.*

STANDARD 3 *Instructional activities for students are based on their needs, interests, and abilities.*

STANDARD 4 *Teaching strategies are matched to the needs of the students and the nature of the objectives.*

STANDARD 5 *The classroom environment is conducive to the learning of mathematics.*

STANDARD 6 *Each teacher maintains and interprets appropriate records of student progress.*

STANDARD 7 *Instructional leadership and support are provided for teachers of mathematics.*

STANDARD 8 *Teachers are involved in the process of organizing for instruction.*

THE CURRICULUM AND INSTRUCTIONAL MATERIALS

CURRICULUM GUIDELINES AND INSTRUCTIONAL MATERIALS ARE A NECESSARY PART OF THE MATHEMATICS PROGRAM. PROGRAM GOALS AND MATERIALS SHOULD REFLECT THE NEEDS OF SOCIAL CHANGE.

STANDARD 9 *Appropriate mathematics curriculum and materials are provided for each student.*

STANDARD 10 *The mathematics program in each school is consistent with state and local goals and objectives.*

STANDARD 11 *An established procedure exists for evaluating school mathematics programs*

STANDARD 12 *The content of the materials (including textbooks) selected to support the program is consistent with the objectives of the mathematics program and students' needs.*

STANDARD 13 *An established procedure exists for the evaluation and selection of mathematics instructional materials within the school.*

STANDARD 14 *Materials are matched with the instructional program to provide for their logical and practical classroom use.*

THE TEACHER OF MATHEMATICS

THE TEACHER OF MATHEMATICS IS THE MOST IMPORTANT COMPONENT OF ANY MATHEMATICS PROGRAM. THE TEACHER MUST HAVE KNOWLEDGE OF MATHEMATICS, UTILIZE A VARIETY OF TEACHING STYLES AND MATERIALS, AND DESIGN APPROPRIATE STRATEGIES TO FACILITATE THE MATHEMATICS EDUCATION OF STUDENTS. MOREOVER, TEACHERS OF MATHEMATICS MUST EXHIBIT POSITIVE ATTITUDES TOWARD MATHEMATICS, SHOW RESPECT FOR "LEARNERS," EXHIBIT ENTHUSIASM FOR TEACHING, AND HAVE A REALISTIC OPINION OF THEIR OWN COMPETENCIES.

STANDARD 15 *A qualified teacher of mathematics is assigned to teach each mathematics class.*

STANDARD 16 *The school administration is committed to continuing teacher education.*

STANDARD 17 *The needs, interests, and desires of teachers of mathematics are considered in preparing inservice education opportunities.*

STANDARD 18 *Teachers of mathematics seek opportunities for professional growth.*

STANDARD 19 *The evaluation of the effectiveness of the mathematics teacher is continuous.*

PHYSICAL FACILITIES AND EQUIPMENT

THE PHYSICAL FACILITIES OF SCHOOL BUILDINGS SHOULD PROMOTE OPTIMUM MATHEMATICS PERFORMANCE FROM EACH STUDENT. WHEN PLANNING A NEW OR RENOVATING AN EXISTING FACILITY, THE FOLLOWING CONSIDERATIONS SHOULD BE INCORPORATED INTO THE DESIGN PHASES OF THE TOTAL FACILITY. SUCCESS IN ADAPTING EXISTING BUILDINGS AND PROGRAMS WILL VARY, DEPENDING ON LOCAL CONDITIONS.

STANDARD 20 *The facility allows for sufficient instructional space for the mathematics program.*

STANDARD 21 *Equipment is provided for the mathematics program.*

Appendix J
Sample Mathematics Textbook Adoption Evaluation Form

Source: *Guidelines for Mathematics Instruction*, 1977, Indiana Department of Public Instruction

CATEGORY: Grade Level _____

Subject _____

TITLE OF BOOK(S) _____

_____

_____ COPYRIGHT DATE _____ 19 ____

AUTHOR(S) _____

_____

PUBLISHER _____

EVALUATOR _____ DATE _____ 19 ____

DIRECTIONS. Read each question slowly and thoroughly. It is important that you read them in their entirety.

To answer a question, mark an 'X', on the scale 0 to 6, to the left of the question between the hash marks over the number indicating the degree or extent you evaluate the material's characteristics.

In addition, you have been given a scale to the right of the question. This scale allows you to grade the value of the question. Please mark an 'X' along the side of the word which indicates the value you would apply to the question.

Finally, after you have marked the above two scales, you may further explain your answer by placing an 'X' alongside the words, "See Narrative." At the end of the form is space for written, subjective answers to clarify or further delineate your response to the question.

/ / / / X / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

1. To what extent does the title indicate the content of the book?

Question is:
__ 0. Unimportant
X 1. Relatively Unimportant
__ 2. Moderately Important
__ 3. Important
__ 4. Extremely Important

ADDITIONAL QUESTIONS, to be answered in the same format as the example, above.

1. To what extent do the materials focus on the major concepts and/or big ideas pertinent to the mathematics category being evaluated?
2. To what extent is the subject matter geared to the interests, abilities, and needs of the students for which the materials are being considered?
3. To what extent can the teacher depart from the sequence of materials prescribed by the author without impairing the effectiveness of the materials?
4. To what extent does the physical and technical (not substantive) quality of the student materials, including all media, meet professional expectations?
5. To what extent does the physical and technical quality of the supportive teacher materials (teacher's guide, media items, etc.) meet professional expectations?
6. To what extent does the language of the textbook accommodate the range of abilities of the students most likely to be using the textbook?
7. To what extent do the materials accommodate the range of reading abilities of the students most likely to be using the materials?
8. To what extent does the text use consistent, acceptable terminology and symbols?
9. To what extent does the material follow a logical, sequential development?
10. To what extent are fundamental principles correctly used throughout the text?
11. To what extent does the text develop concepts and skills in a spiral manner?
12. To what extent does the text provide the student with an opportunity to develop both inductive and deductive reasoning?
13. To what extent are definitions and explanatory materials clear and precise?
14. To what extent does the text present exercises which are adequate to develop the pupil's mastery of the concept being presented?
15. To what extent does the content of the textbook—both the pictorial and written content—reflect the pluralistic, multi-ethnic nature of our society, both past and present?
16. To what extent does the material depict both male and female members of various groups of society in situations which exhibit them unbiasedly?
17. To what extent would the book tend to encourage a positive self-image for children of all ethnic and socioeconomic groups?
18. To what extent does the book develop accurate mathematical concepts and generalizations?

19. To what extent does the author stress the importance of rational thought in discovering and testing the validity of concepts?
20. To what extent are there opportunities for alternative teaching strategies?
21. To what extent would the materials contained in this textbook be learnable and/or teachable?
22. To what extent would you (the reviewer) recommend that these materials be used? *Note*: Question number 22 receives three (3) times the importance of any single preceding question.
23. Should this textbook material be one (1) of the textbooks adopted in this category?
24. If your answer for question number 23 was yes, how would you rank this textbook?
25. Are these materials designed for students with reading disabilities?
26. To what extent does the title indicate the content of the book?

Appendix K
National Council of Teachers of Mathematics Recommendations for the '80s

AN AGENDA FOR ACTION
Recommendations for
School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

Appendix L
National Council of Supervisors of Mathematics Ten Basic Skill Areas

Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

Applying Mathematics to Everyday Situations

The use of mathematics is interrelated with all computation activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.

Alertness to the Reasonableness of Results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and Approximation

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a

particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill. Moreover, there are everyday situations which demand recognition of, and simple computation with, common fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry

Students should learn the geometric concepts they will need to function effectively in the 3-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects.

Measurement

As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Reading, Interpreting, and Constructing Tables, Charts, and Graphs

Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using Mathematics to Predict

Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer Literacy

It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.

Footnotes

1. Robert W. Wirtz, *Mathematics for Everyone* (Washington, DC: Curriculum Development Associates, 1974).

2. Edward P. Labinowicz, *The Piaget Primer: Thinking, Learning, Teaching* (Reading, MA: Addison-Wesley Publishing Co., Inc., 1980).

3. W.W. Sawyer, as quoted in Robert W. Wirtz, *Mathematics for Everyone* (Washington, DC: Curriculum Development Associates, 1974).

4. National Council of Teachers of Mathematics, *An Agenda for Action* (Reston, VA: 1980).

5. Ohio Department of Education, *Problem Solving . . . A Basic Mathematics Goal*, vol. 2 (Columbus, OH: 1980).

6. National Council of Teachers of Mathematics, "Position Statement on Calculators in the Classroom" (Reston, VA: 1978).

7. David C. Johnson, et al., "Computer Literacy—What Is It?", *The Mathematics Teacher*, vol. 73, no. 2 (Reston, VA: National Council of Teachers of Mathematics, February 1980).

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